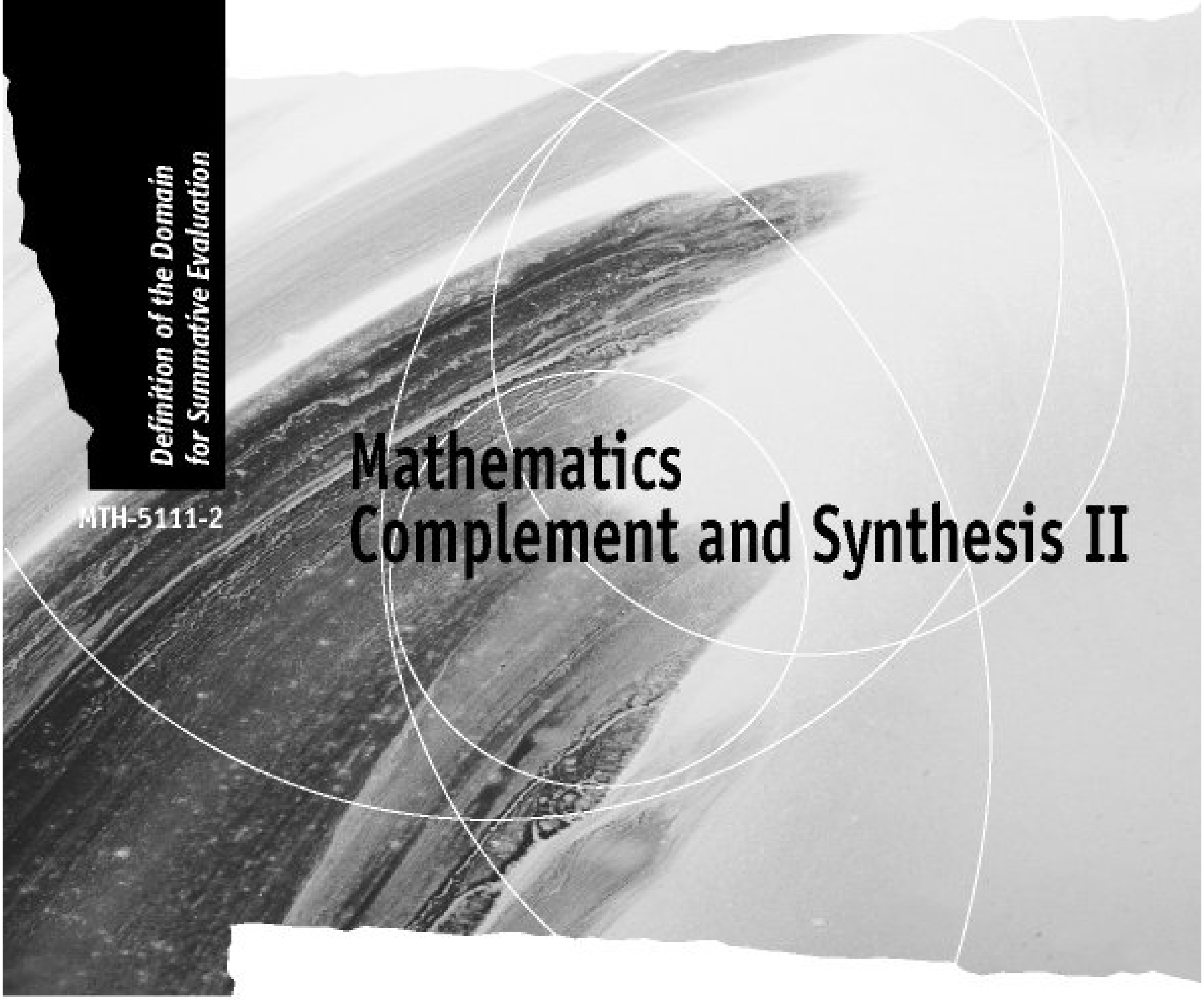
Definition of the Domain for Summative Evaluation

MTH-5111-2

Mathematics Complement and Synthesis II

Québec



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Mathematics Complement and Synthesis II

Formation professionnelle et technique
et formation continue

Direction de la formation générale
des adultes

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1. INTRODUCTION

This Definition of the Domain for Summative Evaluation describes and classifies the essential and representative elements of the secondary-level adult education Mathematics program and, more specifically, of the course entitled Complement and Synthesis II. As such, it gives an overview of the program, but should by no means replace the program itself. The purpose of defining the domain is to ensure that all summative evaluation instruments are consistent with the overall program.

The Definition of the Domain for Summative Evaluation for each course in this program is organized in a similar manner; however, the content of this definition of the domain is specific to the course entitled Complement and Synthesis II.

The goal of the Definition of the Domain for Summative Evaluation is to prepare examinations that are valid from one version to another or from one school board to another, taking into account the responsibilities shared by the Ministère de l'Éducation and the school boards.

2. PROGRAM ORIENTATIONS AND CONSEQUENCES FOR SUMMATIVE EVALUATION

ORIENTATIONS	CONSEQUENCES
The main objective of the secondary-level adult education Mathematics program is to help students fully understand mathematical concepts.	Evaluation should involve verifying whether the student has fully understood the different concepts.
The program is designed to help students master the use of certain mathematical tools used in the field of science and technology or in different trades.	Evaluation items should pertain to situations in the field of science and technology or to situations related to trades.
The program aims to provide students with the skills they need to process information by applying mathematical models and appropriate strategies for solving problems.	Evaluation items should involve performing tasks that require the students to classify information, use mathematical models and solve problems.
The program also aims to improve the students' ability to clearly relate information using mathematical language.	Evaluation items should involve performing tasks that require the use of mathematical language. The appropriateness and clarity of the language used should be taken into account in the marking process.
The program is intended to help students develop a systematic work method.	Evaluation items should require the students to present their work in a clear and structured manner. This should be taken into account in the marking process.
The program will help students master the use of technological tools.	The use of a scientific calculator or graphing calculator is permitted for the examinations related to this course.

3. CONTENT OF THE PROGRAM FOR PURPOSES OF SUMMATIVE EVALUATION

Concepts

Operations on functions and compositions of functions
- graph that represents the result of an operation on two functions
- rule of a composition
- image of elements in compositions
- characteristics of a composition of functions or of the function representing the result of an operation: type of function, domain and range, intervals over which the function is increasing or decreasing, maximum or minimum

Inequalities
- solving inequalities containing absolute values
- solving inequalities containing square roots
- solving second-degree inequalities
- problems involving inequalities containing absolute values
- problems involving inequalities containing square roots
- problems involving second-degree inequalities

Geometry
- completion of a demonstration
- proving a statement using principles pertaining to circles or right triangles
- relationships governing measurements between the elements of a figure
- measures of segments
- problems requiring the application of prior learning

Skills

Each skill is defined within the context of a mathematics program.

Structuring	Being familiar with the fundamentals of mathematics, understanding some mathematical concepts and establishing simple cognitive relations among them. Possible actions: to associate, classify, compare, complete, describe, define, contrast, distinguish, state, enumerate, group, name, rank, organize, recognize, arrange, and so on.
Operating	Performing a given operation or transformation. Possible actions: to calculate, construct, break down, perform, estimate, evaluate, isolate, measure, reconstruct, solve, draw, transform, verify, and so on.
Analyzing	Demonstrating, in an organized fashion, the complex connections between concepts or definitions and their related actions and illustrations. Possible actions: to conclude, correct, deduce, derive, demonstrate, explain, extrapolate, infer, justify, and so on.
Synthesizing	Effectively integrating a variety of concepts and skills to solve a problem. Possible actions: to solve a problem.

4. TABLE OF DIMENSIONS

CONCEPTS SKILLS	OPERATIONS AND COMPOSITIONS 25%	INEQUALITIES 20%	GEOMETRY 55%
OPERATING **35%**	Determine the rules of the compositions of two functions and the image of an element, using the rules of these functions. **1** **10%**	Solve two inequalities containing one real variable that is of degree 2 or that contains an absolute value or a square root. **5** **10%**	Find the measurement of a segment using at least one of principles 80 to 83. The measurements are numerical. **7** **5%**
			Find the measurement of a segment using at least one of principles 80 to 83. The measurements are literal. **8** **10%**
MATHEMATIZING **5%**			Verify the relationships between the elements of a figure. **9** **5%**
ANALYZING **30%**	Determine which statements compare the characteristics of a function with the characteristics of the function representing the result of an operation, given the rules of the two functions. **or** Compare one characteristics of the function representing the result of an operation with that characteristic as it pertains to the initial function. **2** **5%**		
	Determine the operation performed on two functions, given the graphs of these two functions and the graph representing the result of the operation involving these functions. **3** **5%**		Complete a demonstration using principles pertaining to circles or right triangles. **10** **5%**
	Given statements that compare the characteristics of a function with those of a composition function, determine whether the statements are true or false. **4** **5%**		Demonstrate a statement using geometric principles pertaining to circles or right triangles. **11** **10%**
SYNTHESIZING **30%**		Solve a problem involving an inequality containing one real variable that is of degree 2 or that contains an absolute value or a square root. **6** **10%**	Solve two problems involving circles or a right triangles. **12** **20%**

5. OBSERVABLE BEHAVIOURS

Examination items should be formulated on the basis of the observable behaviours listed below. The requirements and restrictions specified in the dimensions and the objectives of the program must be observed.

Dimension 1

Given the rules of two functions, determine the rules of two compositions of these functions. Also determine the image of an element using one of its rules.
(operating) /10

Dimension 2

Given the rules of two real functions and statements that compare certain characteristics of one of these functions with those of the function representing the result of the sum, difference, product or quotient of the two functions, determine which statements are true.

or

Given the rules of different types of functions and an operation to perform, compare one characteristic of the function that represents the result of an operation with that characteristic as it pertains to one of the initial functions. The students must justify their answers.
(analyzing) /5

Dimension 3

Given graphs illustrating the general depiction of two functions and a graph that represents the result of an operation on these two functions, determine the operation that corresponds to the result.
(analyzing) /5

Dimension 4

Given the rules of two functions and statements that compare certain characteristics of one of these functions with those of the composition function, determine whether the statements are true or false.
(analyzing) /5

Dimension 5

Algebraically solve two of the following three types of inequalities:

– second-degree inequality in one real variable
– inequality in one real variable containing an absolute value
– inequality in one real variable containing a square root

For the last two, the variable is in one single term and only within the absolute value or under the square root. The expression containing the variable should be a first-degree expression. The answer should be represented by a numerical line, using set-builder or interval notation. The students must clearly show all their work.

(operating) /10

Dimension 6

Solve a problem involving a second-degree inequality in one real variable, an inequality containing an absolute value or an inequality containing a square root. Solving the problem requires an analysis in order to construct the inequality using the rule of a function given in the statement of the problem. The students must clearly show all their work.

(synthesizing) /10

Note: The inequality must be different from the two inequalities selected for Dimension 5.

Dimension 7

Find the measurement of a segment using at least one of the four principles in numbered 80 to 83. The measurements are given in numerical form and the figure must be included in the statement of the problem. The students must identify the geometric principle used.

(operating) /5

Dimension 8

Find the measurement of a segment using at least one of the four principles in numbers 80 to 83. The principle or principles used must be different from those in the previous dimension. The measurements are given in literal form and the figure must be included in the statement of the problem. The students must clearly show all their work and identify the geometric principles that justify their answer, if applicable.

(operating) /10

Dimension 9

Given a figure whose elements are identified and statements given in symbolic form that describe a relationship governing measurements between the elements, determine whether the statements are true or false.
(mathematizing) /5

Dimension 10

Complete a demonstration using principles pertaining to circles or right triangles.
(analyzing) /5

Dimension 11

Demonstrate a geometric principle pertaining to circles or right triangles. The figure must be included in the statement of the problem. The students must clearly show all their work.
(analyzing) /10

Dimension 12

Solve two problems involving the relationships governing measurements in circles and right triangles that make it possible to apply prior learning related to equations, functions, trigonometric ratios, concepts of analytic geometry and concepts of geometry. The figure must be included in the statement of the problem. The students must clearly show all their work.
(synthesizing) /20

6. JUSTIFICATION OF CHOICES

In the examination, 35% of the items test the students' **OPERATING** skills by verifying whether they have mastered certain operations or transformations:

- solving inequalities
- finding the rule of compositions of given functions
- determining the measurement of segments in a circle or a triangle

In the examination, 5% of the items test the students' **MATHEMATIZING** skills by verifying whether they are able to translate a given situation into a mathematical model:

- verifying the relationships governing measurements between the elements of a figure

In the examination, 30% of the items test the students' skill in **ANALYZING** information; they involve verifying whether the students have the ability to make connections:

- between the characteristics of functions and the characteristics of their composition
- between the characteristics of functions and the characteristics of the result of operations involving these functions
- between the graphs of functions and the graph of the result of operations involving these functions
- by proving principles pertaining to circles and right triangles

In the examination, 30% of the items test the students' **SYNTHESIZING** skills by verifying their ability to:

- solve problems
- use a rigorous work method
- communicate clearly using mathematical language

7. DESCRIPTION OF THE EXAMINATION

A. TYPE OF EXAMINATION

The summative examination will be a written examination consisting of multiple-choice, short-response or extended-response items.

The items should take into account the restrictions and the requirements specified in the dimensions and the objectives of the program. The weighting of marks should be consistent with the percentages set out in the table of dimensions.

B. CHARACTERISTICS OF THE EXAMINATION

All parts of the examination will be administered in a single session lasting no more than three hours.

Students are permitted to use a scientific calculator or a graphing calculator.

A list of geometric principles will be provided (see appendix).

C. PASS MARK

The pass mark is set at 60 out of 100.

APPENDIX

PART I: PRINCIPLES FROM MTH-4111-2
(NUMBERS 1 to 55)

ANGLES

1. Adjacent angles whose external sides are in a straight line are supplementary.
2. Vertically opposite angles are congruent.
3. If a transversal intersects two parallel lines, then:
   a) the alternate interior angles are congruent
   b) the alternate exterior angles are congruent
   c) the corresponding angles are congruent
4. If two corresponding (or alternate interior or alternate exterior) angles are congruent, then they are formed by two parallel lines and a transversal.

TRIANGLES

5. The sum of the measures of the interior angles of a triangle is 180°.
6. In any triangle, the longest side is opposite the largest angle.
7. In any isosceles triangle, the angles opposite the congruent sides are congruent.
8. In any equilateral triangle, each angle measures 60°.
9. In any isosceles triangle, the perpendicular bisector of the side adjacent to the congruent angles is the bisector of the angle opposite this side as well as the median and altitude to this side.
10. In any right triangle, the acute angles are complementary.
11. In any isosceles right triangle, each acute angle measures 45°.
12. In a right triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the other two sides (Pythagorean theorem).
13. A triangle is right-angled if the square of the length of one of its sides is equal to the sum of the squares of the lengths of the other two sides.
14. In a right triangle, the length of the side opposite a 30° angle is equal to half the length of the hypotenuse.
15. Two triangles whose corresponding sides are congruent must be congruent.
16. If two sides and the contained angle of one triangle are congruent to the corresponding sides and contained angle of another triangle, then the triangles must be congruent.

17. If two angles and the contained side of one triangle are congruent to the corresponding angles and contained side of another triangle, then the triangles must be congruent.
18. If two angles of one triangle are congruent to the two corresponding angles of another triangle, then the triangles must be similar.
19. If the lengths of the corresponding sides of two triangles are in proportion, then the triangles must be similar.
20. If the lengths of two sides of one triangle are proportional to the lengths of the two corresponding sides of another triangle and the contained angles are congruent, then the triangles must be similar.
21. In a right triangle, the sine of an acute angle is equal to the ratio obtained by dividing the length of the side opposite this angle by the length of the hypotenuse.

    $\sin A = \frac{a}{c}$, where *a* is the length of the side opposite angle *A* and *c* is the length of the hypotenuse.

22. In a right triangle, the cosine of an acute angle is equal to the ratio obtained by dividing the length of the side adjacent to this angle by the length of the hypotenuse.

    $\cos A = \frac{b}{c}$, where *b* is the length of the side adjacent to angle *A* and *c* is the length of the hypotenuse.

23. In a right triangle, the tangent of an acute angle is equal to the ratio obtained by dividing the length of the side opposite this angle by the length of the side adjacent to it.

    $\tan A = \frac{a}{b}$, where *a* is the length of the side opposite angle *A* and *b* is the length of the side adjacent to angle *A*.

24. The lengths of the sides of any triangle are proportional to the sines of the angles opposite these sides (law of sines):

    $$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

25. The square of the length of a side of any triangle is equal to the sum of the squares of the lengths of the other two sides minus twice the product of the lengths of the other two sides multiplied by the cosine of the contained angle (law of cosines):

    $$a^2 = b^2 + c^2 - 2bc \bullet \cos A$$

    $$b^2 = a^2 + c^2 - 2ac \bullet \cos B$$

    $$c^2 = a^2 + b^2 - 2ab \bullet \cos C$$

QUADRILATERALS

26. The opposite angles of a parallelogram are congruent.
27. The opposite sides of a parallelogram are congruent.
28. The diagonals of a parallelogram bisect each other.

29. The diagonals of a rectangle are congruent.
30. The diagonals of a rhombus are perpendicular to each other.

CIRCLES

31. All the diameters of a circle are congruent.
32. In a circle, the measure of a diameter is equal to twice the measure of the radius.
33. The axes of symmetry of a circle contain its centre.
34. The ratio of the circumference of a circle to its diameter is a constant known as π: $C = \pi d$ or $C = 2\pi r$, where *C* is the circumference, *d* is the diameter and *r* is the radius.
35. The area of a circle is equal to πr^2: $A = \pi r^2$, where *A* is the area and *r* is the radius.

ISOMETRIES AND CONGRUENT FIGURES

36. An isometry preserves collinearity, parallelism, the order of points, distances and the measures of angles. In addition, translations and rotations preserve the orientation of the plane.
37. Any translation will transform a straight line into another line parallel to it.
38. Plane figures or solids are congruent if and only if there is an isometry that maps one figure onto the other.
39. In congruent plane figures or solids, the measures of the following elements are equal:
    a) the corresponding segments and angles
    b) the perimeters
    c) the areas
    d) the volumes
40. Any point on the perpendicular bisector of a segment is equidistant from the two endpoints of this segment.
41. Any point on the bisector of an angle is equidistant from the sides of this angle.
42. In any right triangle, the length of the median to the hypotenuse is equal to half the length of the hypotenuse.
43. The three perpendicular bisectors of the sides of a triangle are concurrent in a point that is equidistant from the three vertices.
44. The diagonals from one vertex of a convex polygon form n - 2 triangles, where n is the number of sides in that polygon.
45. The sum of the measures of the interior angles of a polygon is 180° (n - 2), where n is the number of sides in the polygon.
46. In a convex polygon, the sum of the measures of the exterior angles, one at each vertex, is 360°.

SIMILARITY TRANSFORMATIONS AND SIMILAR FIGURES

47. Any similarity transformation preserves collinearity, parallelism, the order of points, the orientation of the plane, the measures of angles and the ratio of the distances.
48. Any dilatation will transform a straight line into another line parallel to it.
49. Plane figures or solids are similar if and only if there is a similarity transformation that maps one figure onto the other.
50. In similar plane figures or solids:
    a) the ratio of the lengths of the corresponding segments is equal to the scale factor
    b) the ratio of the measures of the corresponding angles is 1
    c) the ratio of the areas is equal to the square of the scale factor
    d) the ratio of the volumes is equal to the cube of the scale factor
51. Plane figures or solids with a scale factor of 1 are congruent.
52. Any straight line that intersects two sides of a triangle and is parallel to the third side forms a smaller triangle similar to the larger triangle.
53. Transversals intersected by parallel lines are divided into segments of proportional lengths.
54. The line segment joining the midpoints of two sides of a triangle is parallel to the third side and its length is one-half the length of the third side.
55. The three medians of a triangle are concurrent in a point that is two-thirds the distance from each vertex to the midpoint of the opposite side.

PART TWO: PRINCIPLES SPECIFIC TO THIS COURSE (NUMBERS 56 to 91)

FUNDAMENTAL PRINCIPLES

56. Two points are in exactly one line.
57. If two lines intersect, they share exactly one point.
58. There is exactly one line that is parallel to a given line and that contains a given point not in the given line.
59. If two lines are parallel to a third line, then they are parallel to each other.
60. If two lines are perpendicular to the same third line, then they are parallel to each other.
61. If a line is perpendicular to one of two parallel lines, then it is perpendicular to the other.
62. For any line and point in a plane, there is exactly one line in the plane that contains the point and is perpendicular to the line.

63. In any triangle, the length of any side is less than the sum of the lengths of the other two sides.
64. In any triangle, the length of any side is greater than the difference of the lengths of the other two sides.
65. If an acute angle and a leg of one right triangle are congruent to an acute angle and the corresponding leg of another right triangle, then the triangles are congruent.
66. If the legs of one right triangle are congruent to the legs of another right triangle, then the triangles are congruent.

CIRCLES

67. Three non-collinear points determine one and only one circle.
68. The diameter is the longest chord of a circle.
69. Any diameter divides a circle into two congruent parts.
70. In a circle or in congruent circles, congruent arcs are subtended by congruent chords, and vice versa.
71. Any diameter perpendicular to a chord divides that chord and each of the arcs that it subtends into two congruent parts. Conversely, any diameter that divides a chord (and each arc that it subtends) into two congruent parts is perpendicular to that chord.
72. In a circle or in two congruent circles, two congruent chords are equidistant from the centre and vice versa.
73. If a line is perpendicular to a radius of a circle at the endpoint of the radius in the circle, the line is tangent to the circle. The converse is also true.
74. Two parallel lines, be they secants or tangents, intercept congruent arcs of a circle.
75. If point *P* is located outside circle *O*, and if segments *PA* and *PB* are tangents to that circle at points *A* and *B* respectively, then OP bisects the angle APB and $\overline{PA} \cong \overline{PB}$.
76. In a circle, the measure of the central angle is equal to the degree measure of its intercepted arc.
77. The measure of an inscribed angle is one-half the measure of its intercepted arc.
78. The measure of an angle formed by two chords intersecting in the interior of a circle is one-half the sum of the measures of the arcs intercepted by the angle and its vertical angle.
79. The measure of an angle formed by two tangents, a tangent and a secant, or two secants is one-half the difference of the measures of the intercepted arcs.
80. In any triangle, the bisector of an angle divides the opposite side into two segments whose lengths are proportional to those of the adjacent sides.

81. **If two chords of a circle intersect in its interior, the product of the lengths of the segments of one chord is equal to the product of the lengths of the segments of the other chord.**

82. **If secants *PAB* and *PCD* of a circle have the same external endpoint *P*, then** $m\overline{PA} \times m\overline{PB} = m\overline{PC} \times m\overline{PD}$**.**

83. **If tangent *PA* and secant *PBC* of a circle have the same external endpoint *P*, then** $\left(m\overline{PA}\right)^2 = m\overline{PB} \times m\overline{PC}$**.**

84. In a circle, the ratio of the measures of two central angles is equal to the ratio of the measures of their intercepted arcs.

85. In a circle, the ratio of the areas of two sectors is equal to the ratio of the measures of their central angles.

86. The circumferences of two circles have the same ratio as their radii.

87. The areas of two circles have the same ratio as the square of their radii.

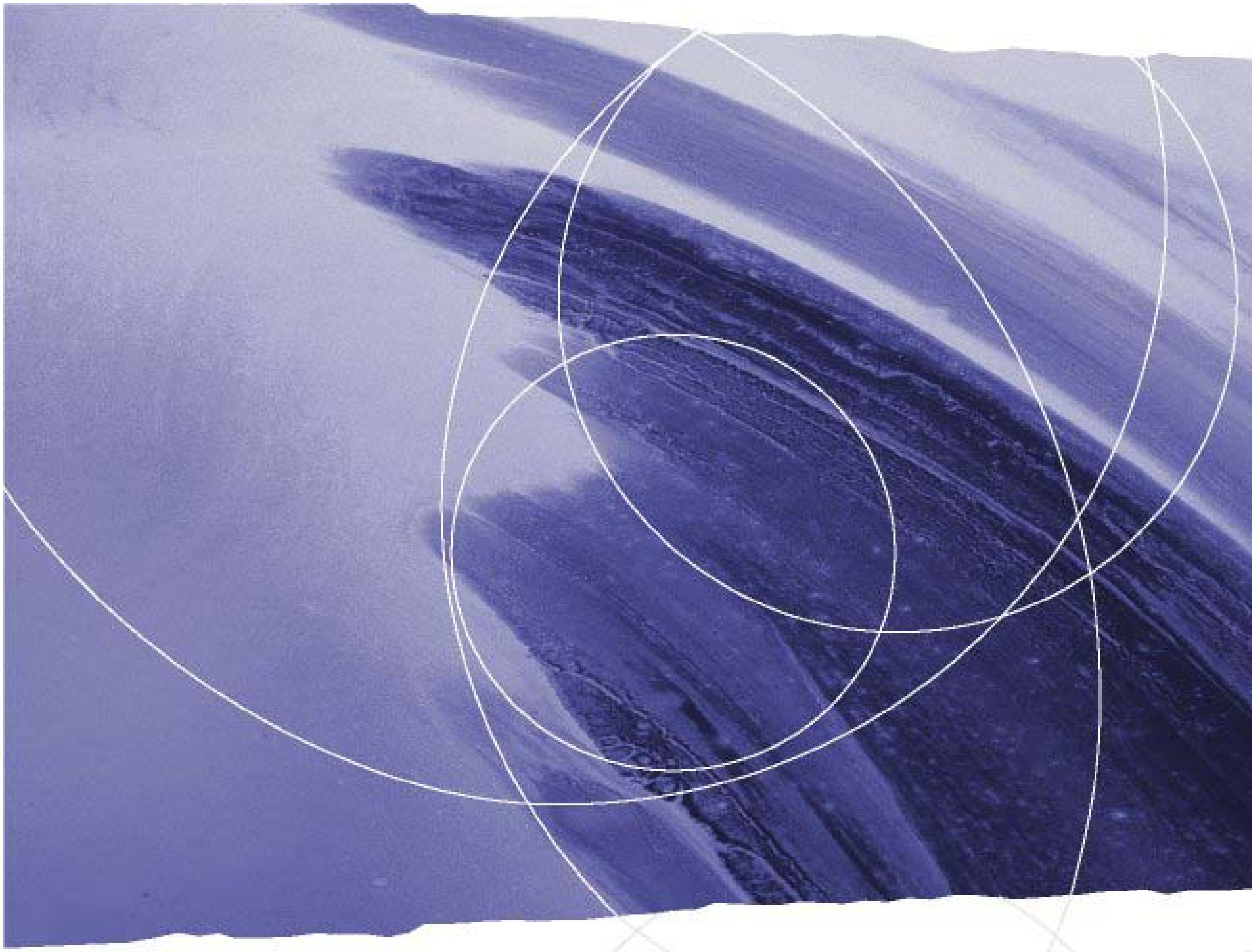
88. The measures of the similar arcs of two circles have the same ratio as their radii.

RIGHT TRIANGLES

89. The length of a leg of a right triangle is the geometric mean between the length of its projection on the hypotenuse and the length of the hypotenuse.

90. The length of the altitude to the hypotenuse of a right triangle is the geometric mean between the lengths of the segments of the hypotenuse.

91. In a right triangle, the length of the hypotenuse multiplied by the length of the altitude to the hypotenuse is equal to the product of the lengths of the sides of the right angle.



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