Mathematics Trigonometric Functions and Equations

Québec

Mathematics
Trigonometric Functions and Equations

1. INTRODUCTION

This Definition of the Domain for Summative Evaluation describes and classifies the essential and representative elements of the secondary-level adult education Mathematics program and, more specifically, of the course entitled Trigonometric Functions and Equations. As such, it gives an overview of the program, but should by no means replace the program itself. The purpose of defining the domain is to ensure that all summative evaluation instruments are consistent with the overall program.

The Definition of the Domain for Summative Evaluation for each course in this program is organized in a similar manner; however, the content of this definition of the domain is specific to the course entitled Trigonometric Functions and Equations.

The goal of the Definition of the Domain for Summative Evaluation is to prepare examinations that are valid from one version to another or from one school board to another, taking into account the responsibilities shared by the Ministère de l'Éducation and the school boards.

2. PROGRAM ORIENTATIONS AND CONSEQUENCES FOR SUMMATIVE EVALUATION

ORIENTATIONS	CONSEQUENCES
The main objective of the secondary-level adult education Mathematics program is to help students fully understand mathematical concepts.	Evaluation should involve verifying whether the student has fully understood the different concepts.
The program is designed to help students master the use of certain mathematical tools used in the field of science and technology or in different trades.	Evaluation items should pertain to situations in the field of science and technology or to situations related to trades.
The program aims to provide students with the skills they need to process information by applying mathematical models and appropriate strategies for solving problems.	Evaluation items should involve performing tasks that require the students to classify information, use mathematical models and solve problems.
The program also aims to improve the students' ability to clearly relate information using mathematical language.	Evaluation items should involve performing tasks that require the use of mathematical language. The appropriateness and clarity of the language used should be taken into account in the marking process.
The program is intended to help students develop a systematic work method.	Evaluation items should require the students to present their work in a clear and structured manner. This should be taken into account in the marking process.
The program will help students master the use of technological tools.	The use of a scientific calculator or graphing calculator is permitted for the examinations related to this course.

3. CONTENT OF THE PROGRAM FOR PURPOSES OF SUMMATIVE EVALUATION

Concepts

Wrapping function
– coordinates of a trigonometric point
– trigonometric angle corresponding to the coordinates of a trigonometric point

Trigonometric functions
– image of a trigonometric angle for a given a trigonometric function
– characteristics of trigonometric functions in $\mathbb{R}$ or over a given interval

Trigonometric equations and identities
– finding the value of a trigonometric function
– solving a first-degree trigonometric equation
– solving a first- or second-degree trigonometric equation that requires factoring
– simplifying a trigonometric expression using formulas that apply to a sum or a difference of two real numbers
– demonstrating a simple trigonometric identity

Sinusoidal functions
– characteristics of a sinusoidal function, given its rule
– rule of a sinusoidal function
– graph of a sinusoidal function
– problems involving sinusoidal functions

Skills

Each skill is defined within the context of a mathematics program.

Structuring	Being familiar with the fundamentals of mathematics, understanding some mathematical concepts and establishing simple cognitive relations among them.
	Possible actions: to associate, classify, compare, complete, describe, define, contrast, distinguish, state, enumerate, group, name, rank, organize, recognize, arrange, and so on.
Operating	Performing a given operation or transformation.
	Possible actions: to calculate, construct, break down, perform, estimate, evaluate, isolate, measure, reconstruct, solve, draw, transform, verify, and so on.
Analyzing	Demonstrating, in an organized fashion, the complex connections between concepts or definitions and their related actions and illustrations.
	Possible actions: to conclude, correct, deduce, derive, demonstrate, explain, extrapolate, infer, justify, and so on.
Synthesizing	Effectively integrating a variety of concepts and skills to solve a problem.
	Possible actions: to solve a problem.

4.   TABLE OF DIMENSIONS

CONCEPTS SKILLS	WRAPPING FUNCTION 10%	TRIGONOMETRIC FUNCTIONS 20%	TRIGONOMETRIC EQUATIONS AND IDENTITIES 35%	SINUSOIDAL FUNCTIONS 35%
STRUCTURING **5%**				Determine characteristics of a sinusoidal function, given its rule. **11** **5%**
OPERATING **50%**	Determine the coordinates of a trigonometric point. **1** **5%**	Determine the image of a trigonometric angle for a given trigonometric function. **3** **5%**	Calculate the value of a trigonometric function using fundamental identities. **6** **5%**	Find the rule of a sinusoidal function. **12** **5%**
	Determine the trigonometric angle that corresponds to the coordinates of a trigonometric point. **2** **5%**	Determine the characteristics of trigonometric functions. **4** **5%**	Solve a first-degree trigonometric equation. **7** **5%**	
			Solve a first- or second-degree trigonometric equation that requires factoring. **8** **5%**	
			Simplify a trigonometric expression using formulas that apply to the sum or the difference of two real numbers or to twice the value of a real number. **9** **10%**	
ANALYZING **25%**		Compare the characteristics of trigonometric functions. **5** **10%**	Prove a simple trigonometric identity. **10** **10%**	Identify the graph of a sinusoidal function, given its characteristics. **13** **5%**
SYNTHESIZING **20%**				Solve two problems involving sinusoidal functions. **14** **20%**

5.   OBSERVABLE BEHAVIOURS

Examination items should be formulated on the basis of the observable behaviours listed below. The requirements and restrictions specified in the dimensions and the objectives of the program must be observed.

Dimension 1

Determine the coordinates of a trigonometric point using the wrapping function. The angle is expressed in radians and is of the form *nπ*, *nπ /2*, *nπ /3*, *nπ /4* or *nπ /6*, and *n* is an integer.
(operating)   /5

Dimension 2

Determine the trigonometric angle that corresponds to the coordinates of a trigonometric point. The angle is expressed in radians over a given interval. The interval is of the form *[nπ, nπ + 2π]*, and *n* is an integer not equal to 0.
(operating)   /5

Dimension 3

Determine the image of a trigonometric angle for a given trigonometric function. The angle is expressed in radians and is of the form *nπ*, *nπ /2*, *nπ /3*, *nπ /4* or *nπ /6*, and *n* is an integer.
(operating)   /5

Dimension 4

Determine the characteristics of two of three sine, cosine or tangent functions in $\mathbb{R}$ or over a given interval.
(operating)   /5

Dimension 5

Compare three or four characteristics of sine, cosine or tangent functions in $\mathbb{R}$ or over a given interval.
(analyzing)   /10

Dimension 6

Given the value of a trigonometric function at one point within a designated interval, calculate the value of another trigonometric function at this point using fundamental trigonometric identities. The interval measures no more than π radians and is limited to multiples of $\pi/2$.
(operating)   /5

Dimension 7

Solve a first-degree trigonometric equation in $\mathbb{R}$. Solving the equation does not require factoring.
(operating)   /5

Dimension 8

Solve a second-degree trigonometric equation, over a given interval limited to multiples of π. Solving the equation requires only one factorization (finding the common factor, the difference of squares or the trinomial of the form $ax^2 + bx + c$).

The equation must have at least one solution over the designated interval. The students must clearly show all their work.
(operating)   /5

Dimension 9

Simplify a trigonometric expression using the formulas of sine, cosine or tangent functions (the formulas are provided) that apply to the sum or the difference of two real numbers or to twice the value of a real number. The expression must include no more than three trigonometric functions. The students must clearly show all their work.

(When applying formulas for sine or cosine functions, *A* or *B* are multiples of π /2 or a variable. When applying formulas for tangent functions, *A* or *B* are multiples of π /4 or a variable.)
(operating)   /10

Dimension 10

Prove a simple trigonometric identity. The expression should consist of no more than two terms on each side of the equality. Each term should contain no more than two trigonometric functions. The students must clearly show all their work.
(analyzing)   /10

Dimension 11

Given the rule of a sinusoidal function, determine certain characteristics of this function.
(structuring)   /5

Dimension 12

Determine the rule of a sinusoidal function, given pertinent information or the graph of the function.
(operating)   /5

Dimension 13

Given the description of certain characteristics of a sinusoidal function and four graphs, identify the graph of the function described. The rule is not given.
(analyzing)   /5

Dimension 14

Solve two problems involving sinusoidal functions. Solving the problems may require describing certain characteristics of the function, describing the connections between the change in the rule and the transformation of the graph or comparing certain characteristics of two or three sinusoidal functions over a given interval. Solving one of the problems requires finding the rule of the function. The students must clearly show all their work.
(synthesizing)   /20

6. JUSTIFICATION OF CHOICES

In the examination, 5% of the items test the students' **STRUCTURING** skills by verifying their understanding of certain concepts:
– determining the characteristics of a sinusoidal function, given its rule

In the examination, 50% of the items test the students' **OPERATING** skills by verifying whether they have mastered certain operations or transformations:
– determining the coordinates of a trigonometric point
– determining the trigonometric angle corresponding to the coordinates of a trigonometric point
– determining the image of a trigonometric angle for a given trigonometric function
– calculating the value of a trigonometric function
– solving a first-degree equation
– solving a first- or second-degree equation that requires factoring
– simplifying a trigonometric expression
– finding the rule of a sinusoidal function

In the examination, 25% of the items test the students' skill in **ANALYZING** information; they involve verifying whether the students have the ability to make connections:
– between the characteristics of a sinusoidal function and its graph
– between the characteristics of different functions
– by proving a simple trigonometric identity

In the examination, 20% of the items test the students' **SYNTHESIZING** skills by verifying their ability to:
– solve problems
– use a rigorous work method
– communicate clearly using mathematical language

7. DESCRIPTION OF THE EXAMINATION

A. TYPE OF EXAMINATION

The summative examination will be a written examination consisting of multiple-choice, short-response or extended-response items.

The items should take into account the restrictions and the requirements specified in the dimensions and the objectives of the program. The weighting of marks should be consistent with the percentages set out in the table of dimensions.

B. CHARACTERISTICS OF THE EXAMINATION

All parts of the examination will be administered in a single session lasting no more than two and a half hours.

Students are permitted to use a scientific calculator or a graphing calculator.

C. PASS MARK

The pass mark is set at 60 out of 100.