Definition of the Domain for Summative Evaluation

MTH-4111-2

Mathematics Complement and Synthesis I

Québec

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Mathematics
Complement and Synthesis I

Formation professionnelle et technique
et formation continue

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des adultes

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1. INTRODUCTION

This Definition of the Domain for Summative Evaluation describes and classifies the essential and representative elements of the secondary-level adult education Mathematics program and, more specifically, of the course entitled Complement and Synthesis I. As such, it gives an overview of the program, but should by no means replace the program itself. The purpose of defining the domain is to ensure that all summative evaluation instruments are consistent with the overall program.

The Definition of the Domain for Summative Evaluation for each course in this program is organized in a similar manner; however, the content of this definition of the domain is specific to the course entitled Complement and Synthesis I.

The goal of the Definition of the Domain for Summative Evaluation is to prepare examinations that are valid from one version to another or from one school board to another, taking into account the responsibilities shared by the Ministère de l'Éducation and the school boards.

2. PROGRAM ORIENTATIONS AND CONSEQUENCES FOR SUMMATIVE EVALUATION

ORIENTATIONS	CONSEQUENCES
The main objective of the secondary-level adult education Mathematics program is to help students fully understand mathematical concepts.	Evaluation should involve verifying whether the student has fully understood the different concepts.
The program is designed to help students master the use of certain mathematical tools used in the field of science and technology or in different trades.	Evaluation items should pertain to situations in the field of science and technology or to situations related to trades.
The program aims to provide students with the skills they need to process information by applying mathematical models and appropriate strategies for solving problems.	Evaluation items should involve performing tasks that require the students to classify information, use mathematical models and solve problems.
The program also aims to improve the students' ability to clearly relate information using mathematical language.	Evaluation items should involve performing tasks that require the use of mathematical language. The appropriateness and clarity of the language used should be taken into account in the marking process.
The program is intended to help students develop a systematic work method.	Evaluation items should require the students to present their work in a clear and structured manner. This should be taken into account in the marking process.
The program will help students master the use of technological tools.	The use of a scientific calculator or graphing calculator is permitted for the examinations related to this course.

3. CONTENT OF THE PROGRAM FOR PURPOSES OF SUMMATIVE EVALUATION

Concepts

Systems of equations in two variables (one of the equations is of degree 0 or 1 and the other is of degree 2)
- solving a system of equations algebraically
- solving a problem involving a system of equations in two variables

Operations involving polynomial functions
- adding, subtracting, multiplying
- identifying the graph that represents the result of an operation involving the graphs of two functions
- identifying the graph that represents the result of an operation involving two functions described by parametric equations

Analytic geometry
- calculating the area of a triangle, given the coordinates of its vertices
- calculating the area of a quadrilateral, given the coordinates of its vertices (the quadrilateral is not a right quadrilateral)
- finding the equation of a significant line of a triangle, given the coordinates of the triangle's vertices
- proving that a quadrilateral belongs to a specific category
- completing a proof using analytic geometry

Geometry
- proof using the concepts of isometry and similarity
- problem involving similar plane figures
- problem involving similar solids
- problem involving equivalent plane figures
- problem involving equivalent solids

Skills

Each skill is defined within the context of a mathematics program.

Operating	Performing a given operation or transformation. Possible actions: to calculate, construct, break down, perform, estimate, evaluate, isolate, measure, reconstruct, solve, draw, transform, verify, and so on.
Analyzing	Demonstrating, in an organized fashion, the complex connections between concepts or definitions and their related actions and illustrations. Possible actions: to conclude, correct, deduce, derive, demonstrate, explain, extrapolate, infer, justify, and so on.
Synthesizing	Effectively integrating a variety of concepts and skills to solve a problem. Possible actions: to solve a problem.

4. TABLE OF DIMENSIONS

CONCEPTS / SKILLS	SYSTEMS OF EQUATIONS 15%	OPERATIONS INVOLVING FUNCTIONS 10%	ANALYTIC GEOMETRY 30%	GEOMETRY 45%
OPERATING 20%	Algebraically solve a system of equations in two variables. One of the equations is of degree 2. **1** **5%**		Calculate the area of a triangle or a quadrilateral. (The quadrilateral is not a right quadrilateral.) **5** **10%**	
			Determine the equation of a significant line of a triangle. **6** **5%**	
ANALYZING 30%		Identify the graph that represents the result of an operation involving the graphs of two functions. **3** **5%**	Prove that a quadrilateral belongs to a specific category of quadrilaterals. **7** **10%**	
		Identify the graph that represents the result of an operation involving two functions described by parametric equations. **4** **5%**	Complete a proof of a proposition related to a triangle or a quadrilateral using analytic geometry. **8** **5%**	Complete a proof using the concepts of isometry and similarity. **9** **5%**
SYNTHESIZING 50%	Algebraically solve a problem involving a system of equations in two variables. One of the equations is of degree 2. The equations are not given. **2** **10%**			Solve a problem involving similar plane figures. **10** **10%**
				Solve a problem involving similar solids. **11** **10%**
				Solve a problem involving equivalent plane figures. **12** **10%**
				Solve a problem involving equivalent solids. **13** **10%**

5. OBSERVABLE BEHAVIOURS

Examination items should be formulated on the basis of the observable behaviours listed below. The requirements and restrictions specified in the dimensions and the objectives of the program must be observed.

Dimension 1

Algebraically solve a system of two equations in two variables. One of the equations is of degree 0 or 1 and the other is of degree 2. There may be no solution or one or two solutions. The students must clearly show all their work.
(operating) /5

Dimension 2

Algebraically solve a problem involving a system of two equations in two variables. One of the equations is of degree 0 or 1 and the other is of degree 2. Solving the problem requires finding the equations, given the coordinates of certain points provided in a statement that includes a figure or a graph. It also requires performing a comparative analysis or determining the points of intersection of the system. The students must clearly show all their work.
(synthesizing) /10

Dimension 3

Given graphs sketched on paper without any underlying grid of two polynomial functions of a degree less than 3, determine which graph corresponds to the sum, the difference or the product of these functions. The product of the functions must be of a degree less than 3.
(analyzing) /5

Dimension 4

Given parametric equations of two polynomial functions of a degree less than 3 and certain specifications about the value of the parameters, determine which graph corresponds to the sum, the difference or the product of these two functions.
(analyzing) /5

Dimension 5

Calculate the area of a triangle or a quadrilateral, given the coordinates of its vertices. The coordinates must be integers. The triangle is not a right triangle and the quadrilateral is not a right quadrilateral. The students must clearly show all their work.
(operating) /10

Dimension 6

Given the coordinates of its vertices, determine the equation of a significant line of a triangle (altitude, median or perpendicular bisector). The coordinates of the vertices are integers. The students must clearly show all their work.
(operating) /5

Dimension 7

Given the coordinates of its vertices, prove that a quadrilateral belongs to a specific category of quadrilaterals. The coordinates of the vertices are integers. The students must clearly show all their work using the definitions or properties of figures, various formulas or various principles of analytic geometry.
(analyzing) /10

Dimension 8

Using analytic geometry, complete a proof of a proposition related to a triangle or a quadrilateral. In the given figure, the base is on the *x*-axis, and the coordinates of the vertices are literal, except for the two vertices located on the *x*-axis, one of which is located at the origin.
(analyzing) /5

Dimension 9

Complete a proof using the concepts of isometry and similarity. The figure is given.
(analyzing) /5

Dimension 10

Solve a problem involving similar plane figures. The scale factor may be given or determined by comparing lengths or areas. Solving the problem may require students to call upon their knowledge of trigonometry. The students must clearly show all their work.
(synthesizing) /10

Dimension 11

Solve a problem involving similar solids. The scale factor may be given or determined by comparing lengths, areas or volumes. Solving the problem may require students to call upon their knowledge of trigonometry. The students must clearly show all their work.
(synthesizing) /10

Dimension 12

Solve a problem involving equivalent plane figures. Solving the problem may require students to use second-degree equations or call upon their knowledge of trigonometry. The students must clearly show all their work.
(synthesizing) /10

Dimension 13

Solve a problem involving equivalent solids. Solving the problem may require students to use second-degree equations or call upon their knowledge of trigonometry. The students must clearly show all their work.
(synthesizing) /10

Notes: Trigonometry concepts must be used in at least one item from dimensions 10, 11, 12 or 13.

A second-degree equation must be used in one item from dimensions 12 or 13.

6. JUSTIFICATION OF CHOICES

In the examination, 20% of the items test the students' **OPERATING** skills by verifying whether they have mastered certain operations or transformations:

– solving systems of two equations in two variables (one of the equations is a second-degree equation)
– calculating the area of a triangle or a quadrilateral (the quadrilateral is not a right quadrilateral)
– determining the equation of a significant line of a triangle

In the examination, 30% of the items test the students' skill in **ANALYZING** information; they involve verifying whether the students have the ability to make connections:

– by identifying the graph of the function resulting from an operation performed on two functions
– by proving that a quadrilateral belongs to a specific category of quadrilaterals
– by completing a proof using analytical geometry
– by completing a proof using the concepts of isometry and similarity

In the examination, 50% of the items test the students' **SYNTHESIZING** skills by verifying their ability to:

– solve problems
– use a rigorous work method
– communicate clearly using mathematical language

7. DESCRIPTION OF THE EXAMINATION

A. TYPE OF EXAMINATION

The summative examination will be a written examination consisting of multiple-choice, short-response or extended-response items.

The items should take into account the restrictions and the requirements specified in the dimensions and the objectives of the program. The weighting of marks should be consistent with the percentages set out in the table of dimensions.

B. CHARACTERISTICS OF THE EXAMINATION

All parts of the examination will be administered in a single session lasting no more than three hours.

Students are permitted to use a scientific calculator or a graphing calculator, as well as a ruler and a compass or a set-square.

Students will be given a list of formulas and list of principles of geometry (see appendix).

C. PASS MARK

The pass mark is set at 60 out of 100.

Analytic geometry formulas

Distance between a point and a line

$$\frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}} \quad \text{or} \quad \frac{|ax_1 - y_1 + b|}{\sqrt{a^2 + 1}}$$

Point of division of a line segment

$$x = \frac{bx_1 + ax_2}{a+b}, \quad y = \frac{by_1 + ay_2}{a+b}$$

or

$$x = \frac{nx_1 + mx_2}{n+m}, \quad y = \frac{ny_1 + my_2}{n+m}$$

or

$$x = x_1 + \frac{a}{b} \bullet (x_2 - x_1), \quad y = y_1 + \frac{a}{b} \bullet (y_2 - y_1)$$

Distance between two points

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Geometry formulas

Perimeter and area of plane figures

Perimeter = sum of all sides

Area of triangles: $\mathrm{A} = \frac{\mathrm{b} \times \mathrm{h}}{2}$	**Area of trapezoids:** $A = \frac{(\mathrm{B} + \mathrm{b}) \times \mathrm{h}}{2}$
Area of parallelograms: $A = b \times h$	**Area of rhombuses:** $A = \frac{D \times d}{2}$

Solids

SOLID	Formula to calculate lateral area	Formula to calculate total area	Formula to calculate volume
CUBE	$A = 4\,c^2$	$A = 6\,c^2$	$V = c^3$ or V = *(area of base) (height)*
RIGHT PRISM	$A = 2h\,(L + l)$	$A = 2(hL + hl + Ll)$	$V = L \times l \times h$ or V = *(area of base) (height)*
CYLINDER	$A = 2\pi rh$	$A = 2\pi r(h + r)$	$V = \pi\, r^2 h$ or V = *(area of base) (height)*
CONE	$A = \pi\, r\, g$	$A = \pi\, r\,(g + r)$	$V = \frac{1}{3}\pi\, r^2 h$ or $V = \frac{1}{3}$ *(area of base) (height)*

APPENDIX

PART ONE: PRINCIPLES LEARNED IN PREVIOUS COURSES (NUMBERS 1 to 35)

ANGLES

1. Adjacent angles whose external sides are in a straight line are supplementary.
2. Vertically opposite angles are congruent.
3. If a transversal intersects two parallel lines, then:
   a) the alternate interior angles are congruent
   b) the alternate exterior angles are congruent
   c) the corresponding angles are congruent
4. If two corresponding (or alternate interior or alternate exterior) angles are congruent, then they are formed by two parallel lines and a transversal.

TRIANGLES

5. The sum of the measures of the interior angles of a triangle is 180°.
6. In any triangle, the longest side is opposite the largest angle.
7. In any isosceles triangle, the angles opposite the congruent sides are congruent.
8. In any equilateral triangle, each angle measures 60°.
9. In any isosceles triangle, the perpendicular bisector of the side adjacent to the congruent angles is the bisector of the angle opposite this side as well as the median and altitude to this side.
10. In any right triangle, the acute angles are complementary.
11. In any isosceles right triangle, each acute angle measures 45°.
12. In a right triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the other two sides (Pythagorean theorem).
13. A triangle is right-angled if the square of the length of one of its sides is equal to the sum of the squares of the lengths of the other two sides.
14. In a right triangle, the length of the side opposite a 30° angle is equal to half the length of the hypotenuse.
15. Two triangles whose corresponding sides are congruent must be congruent.
16. If two sides and the contained angle of one triangle are congruent to the corresponding sides and contained angle of another triangle, then the triangles must be congruent.
17. If two angles and the contained side of one triangle are congruent to the corresponding angles and contained side of another triangle, then the triangles must be congruent.
18. If two angles of one triangle are congruent to the two corresponding angles of another triangle, then the triangles must be similar.
19. If the lengths of the corresponding sides of two triangles are in proportion, then the triangles must be similar.

20. If the lengths of two sides of one triangle are proportional to the lengths of the two corresponding sides of another triangle and the contained angles are congruent, then the triangles must be similar.

21. In a right triangle, the sine of an acute angle is equal to the ratio obtained by dividing the length of the side opposite this angle by the length of the hypotenuse.

$\sin A = \frac{a}{c}$, where *a* is the length of the side opposite angle *A* and *c* is the length of the hypotenuse.

22. In a right triangle, the cosine of an acute angle is equal to the ratio obtained by dividing the length of the side adjacent to this angle by the length of the hypotenuse.

$\cos A = \frac{b}{c}$, where *b* is the length of the side adjacent to angle *A* and *c* is the length of the hypotenuse.

23. In a right triangle, the tangent of an acute angle is equal to the ratio obtained by dividing the length of the side opposite this angle by the length of the side adjacent to it.

$\tan A = \frac{a}{b}$, where *a* is the length of the side opposite angle *A* and *b* is the length of the side adjacent to angle *A*.

24. The lengths of the sides of any triangle are proportional to the sines of the angles opposite these sides (law of sines):

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

25. The square of the length of a side of any triangle is equal to the sum of the squares of the lengths of the other two sides minus twice the product of the lengths of the other two sides multiplied by the cosine of the contained angle (law of cosines):

$$a^2 = b^2 + c^2 - 2bc \bullet \cos A$$

$$b^2 = a^2 + c^2 - 2ac \bullet \cos B$$

$$c^2 = a^2 + b^2 - 2ab \bullet \cos C$$

QUADRILATERALS

26. The opposite angles of a parallelogram are congruent.
27. The opposite sides of a parallelogram are congruent.
28. The diagonals of a parallelogram bisect each other.
29. The diagonals of a rectangle are congruent.
30. The diagonals of a rhombus are perpendicular to each other.

CIRCLES

31. All the diameters of a circle are congruent.
32. In a circle, the measure of a diameter is equal to twice the measure of the radius.
33. The axes of symmetry of a circle contain its centre.

34. The ratio of the circumference of a circle to its diameter is a constant known as π: $C = \pi d$ or $C = 2\pi r$, where C is the circumference, d is the diameter and r is the radius.
35. The area of a circle is equal to πr^2: $A = \pi r^2$, where A is the area and r is the radius.

PART TWO: PRINCIPLES OF GEOMETRY LEARNED IN THIS COURSE (NUMBERS 36 TO 55)

ISOMETRIES AND CONGRUENT FIGURES

36. An isometry preserves collinearity, parallelism, the order of points, distances and the measures of angles. In addition, translations and rotations preserve the orientation of the plane.
37. Any translation will transform a straight line into another line parallel to it.
38. Plane figures or solids are congruent if and only if there is an isometry that maps one figure onto the other.
39. In congruent plane figures or solids, the measures of the following elements are equal:
    a) the corresponding segments and angles
    b) the perimeters
    c) the areas
    d) the volumes
40. Any point on the perpendicular bisector of a segment is equidistant from the two endpoints of this segment.
41. Any point on the bisector of an angle is equidistant from the sides of this angle.
42. In any right triangle, the length of the median to the hypotenuse is equal to half the length of the hypotenuse.*
43. The three perpendicular bisectors of the sides of a triangle are concurrent in a point that is equidistant from the three vertices.
44. The diagonals from one vertex of a convex polygon form n - 2 triangles, where n is the number of sides in that polygon.
45. The sum of the measures of the interior angles of a polygon is 180° (n - 2), where n is the number of sides in the polygon.
46. In a convex polygon, the sum of the measures of the exterior angles, one at each vertex, is 360°.

SIMILARITY TRANSFORMATIONS AND SIMILAR FIGURES

47. Any similarity transformation preserves collinearity, parallelism, the order of points, the orientation of the plane, the measures of angles and the ratio of the distances.
48. Any dilatation will transform a straight line into another line parallel to it.

* Proving this statement also involves applying the principles associated with similar triangles.

49. Plane figures or solids are similar if and only if there is a similarity transformation that maps one figure onto the other.

50. In similar plane figures or solids:
   a) the ratio of the lengths of the corresponding segments is equal to the scale factor
   b) the ratio of the measures of the corresponding angles is 1
   c) the ratio of the areas is equal to the square of the scale factor
   d) the ratio of the volumes is equal to the cube of the scale factor

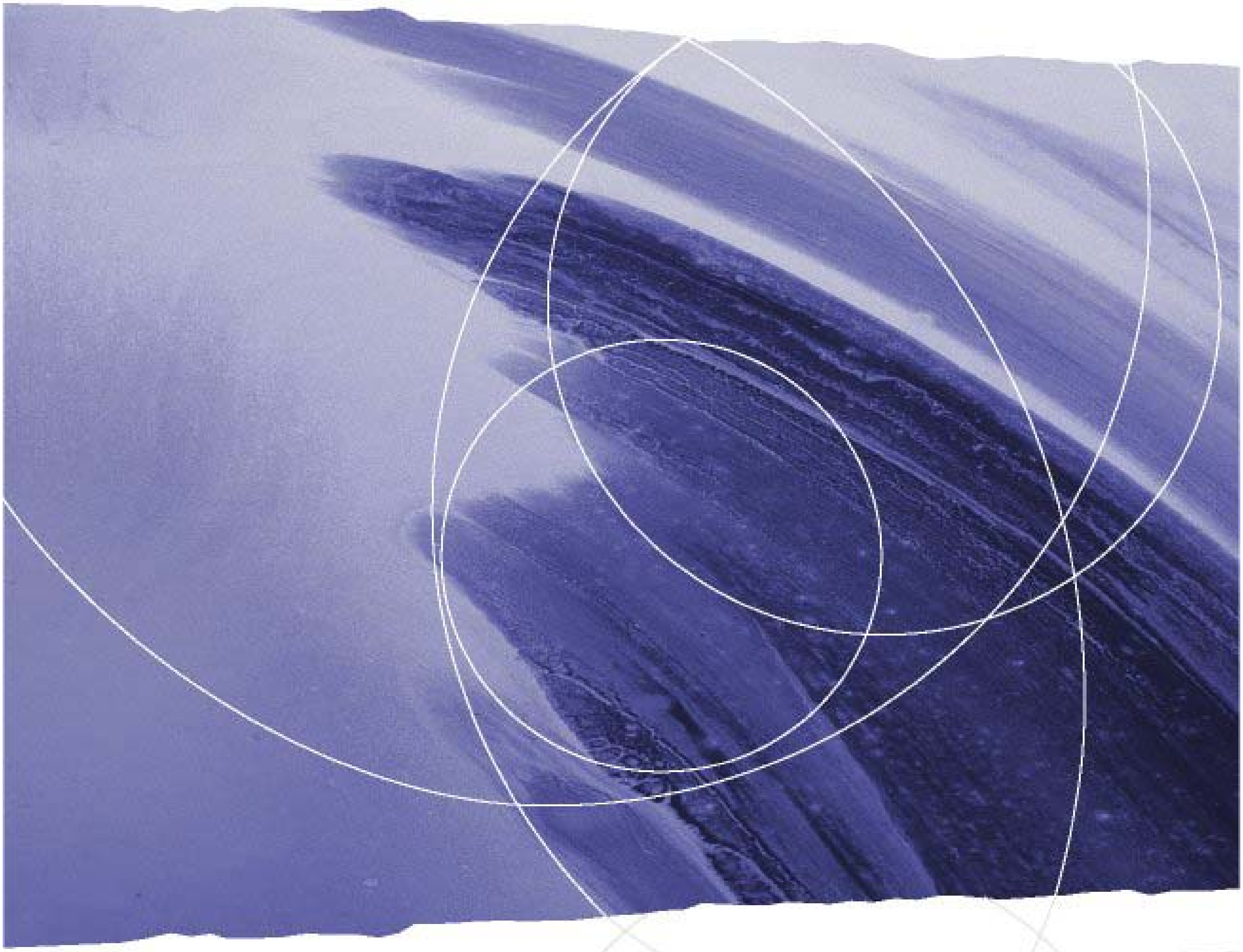
51. Plane figures or solids with a scale factor of 1 are congruent.

52. Any straight line that intersects two sides of a triangle and is parallel to the third side forms a smaller triangle similar to the larger triangle.

53. Transversals intersected by parallel lines are divided into segments of proportional lengths.

54. The line segment joining the midpoints of two sides of a triangle is parallel to the third side and its length is one-half the length of the third side.

55. The three medians of a triangle are concurrent in a point that is two-thirds the distance from each vertex to the midpoint of the opposite side.



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