

MEANING OF OPERATIONS

**Using different meanings of addition,
subtraction, multiplication and division**

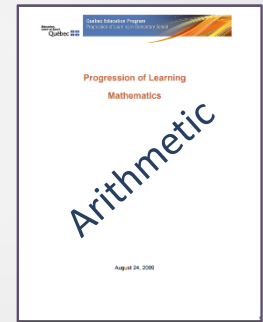
2017-2018

Direction de la formation générale des jeunes
Secteur de l'éducation préscolaire et de l'enseignement primaire et secondaire
Ministère de l'Éducation et de l'Enseignement supérieur

Objectives

- Specify the elements of the different situations presented in the section about the **meaning of operations** in the Progression of Learning.
- Give examples involving different mathematical models.
- Make connections with the different operations.

Progression of Learning: Arithmetic



Understanding and writing numbers

Natural numbers less than . . .
Fractions (using objects or drawings)
Decimals up to . . .
Integers

Meaning of operations

Natural numbers less than . . .
Decimals up to . . .
Fractions

Operations

Natural numbers (based on the benchmarks for each cycle)
Fractions (using objects or diagrams)
Decimals
Using numbers

Meaning of operations

Students will thus be encouraged to use concrete, semi-concrete or symbolic means to mathematize a variety of situations illustrating different meanings.
(Progression of Learning, p. 9)

To mathematize

“To mathematize, one sees, organizes and interprets the world through and with mathematical models.”

Twomey Fosnot, Catherine, and Maarten Dolk, *Young Mathematicians at Work: Constructing Number Sense, Addition, and Subtraction* (Portsmouth, New Hampshire: Heinemann, 2001).

From action...

“When children attempt to model a situation mathematically, they often begin by modeling the actions in the situation.”

Twomey Fosnot, Catherine, and Maarten Dolk, *Young Mathematicians at Work: Constructing Number Sense, Addition, and Subtraction* (Portsmouth, New Hampshire: Heinemann, 2001).

...to representation

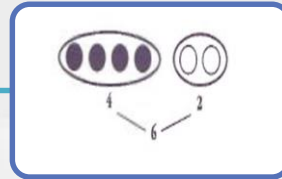
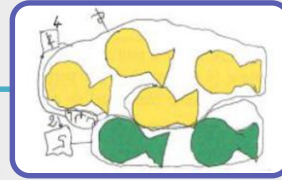
CONCRETE

Manipulate
concrete objects
Manipulate
symbolic objects
(tokens, cubes, etc.)



SEMI-CONCRETE

Pictorial
representation
Figurative*
Pictorial
representation
Simplified*



SYMBOLIC

Symbolic
Representation

$$4 \xrightarrow{+2}$$

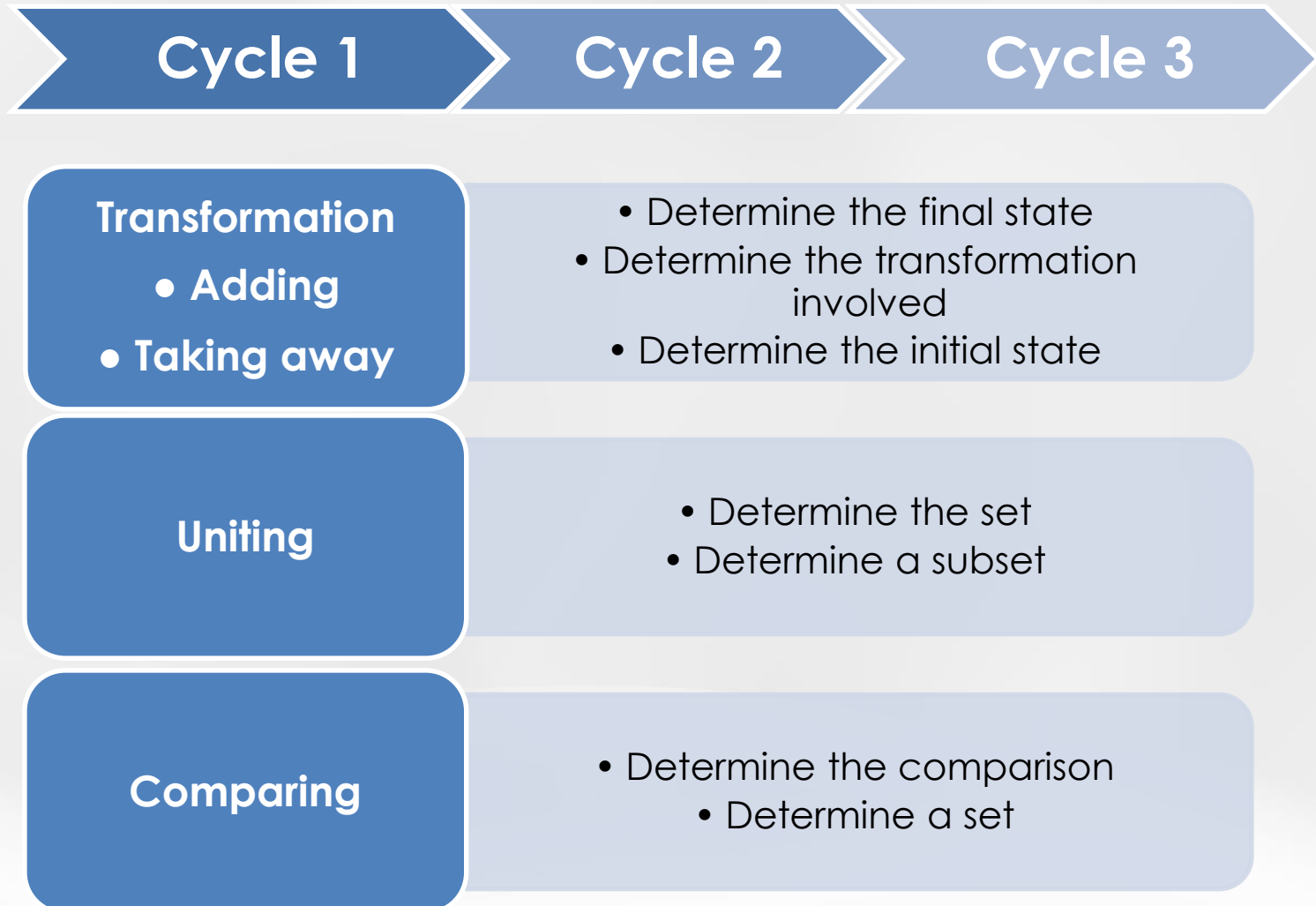
$$4 + 2 = \square$$

MATHEMATICAL MODELS

How can teachers take their students a step further in their representations?



Situations involving additive structures



Situations involving additive structures

Cycle 2

Cycle 3

**Composition of transformations:
positive, negative**

- Determine the gain or the loss
- Determine the transformation involved

I show my students a variety of situations.

Cycle 3

**Composition of transformations:
mixed**

- Determine the gain or loss
- Determine the transformation involved



➔ Situations involving additive structures

Cycles 1, 2 and 3

Transformation

Transformation

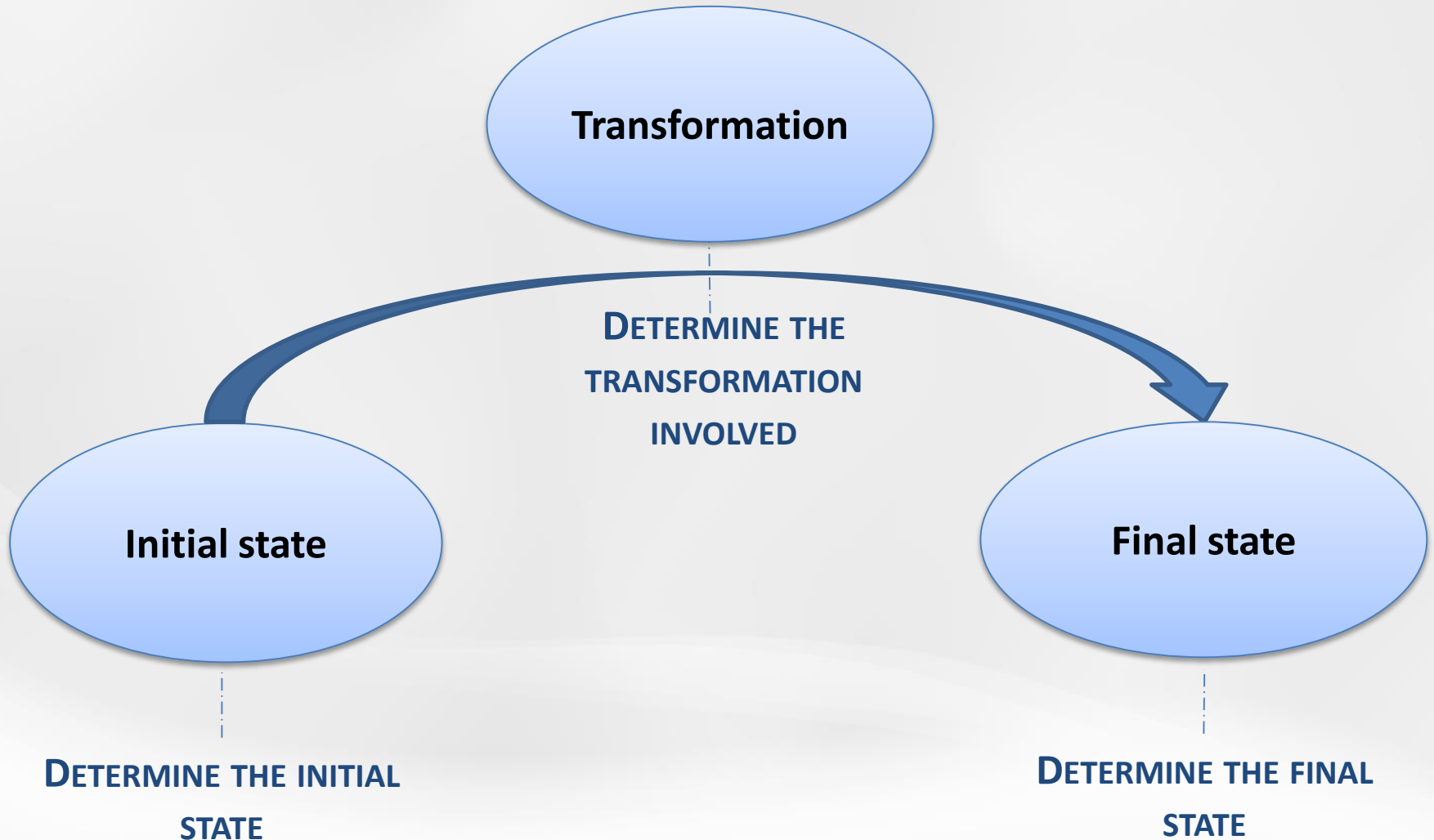
**DETERMINE THE
TRANSFORMATION
INVOLVED**

Initial state

**DETERMINE THE INITIAL
STATE**

Final state

**DETERMINE THE FINAL
STATE**

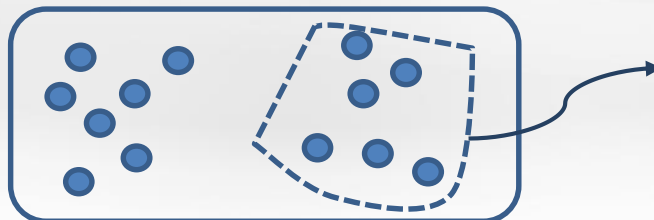


➔ Situations involving additive structures

Transformation

(DETERMINE THE INITIAL STATE)

Gus had a certain number of objects.
He gave 6 to Melanie. He now has 7 objects.
How many objects did Gus start with?

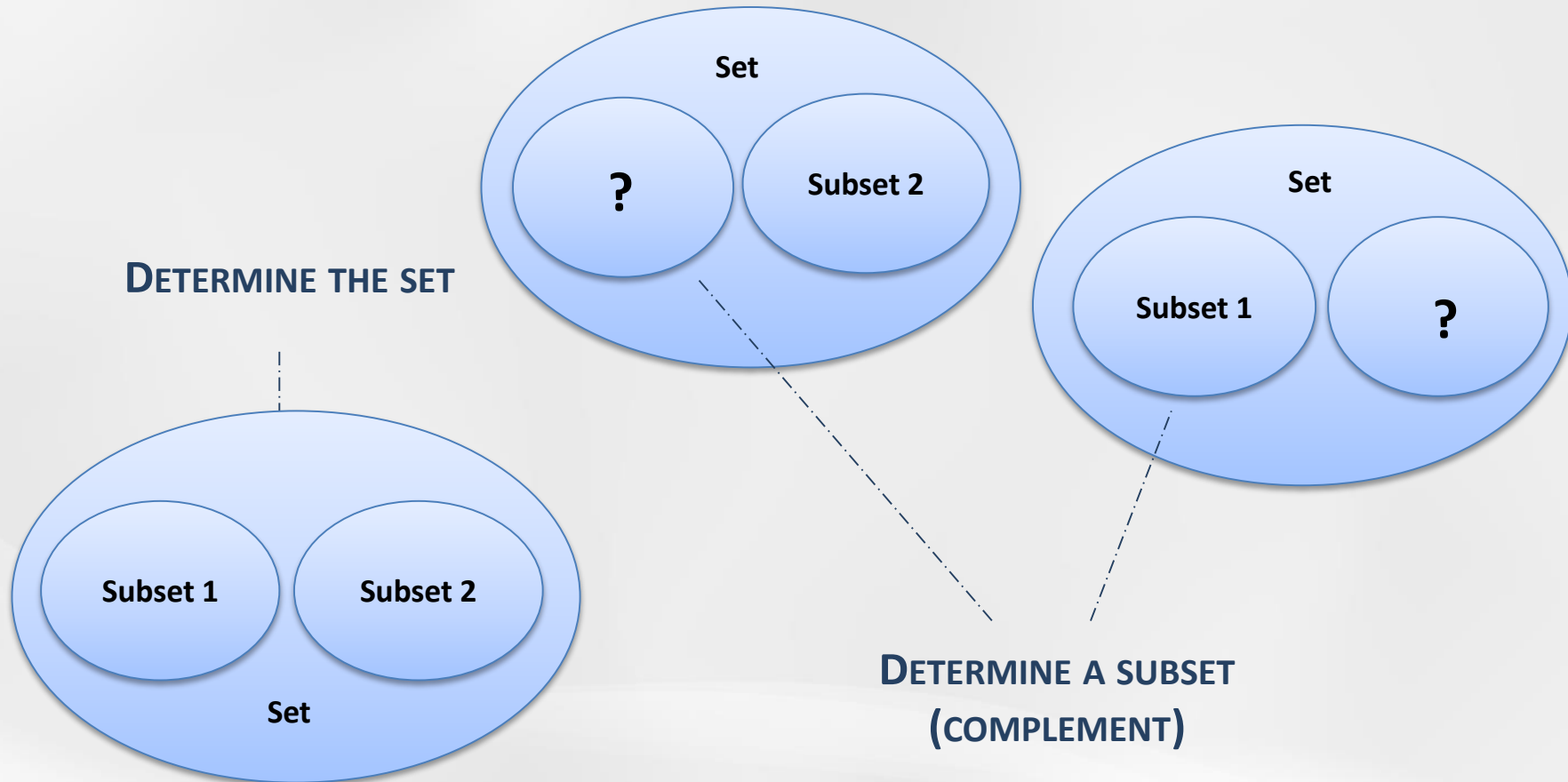


$$\square - 6 = 7$$

➔ Situations involving additive structures

Cycles 1, 2 and 3

Uniting



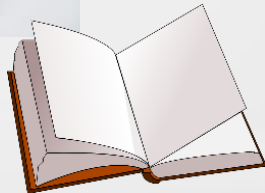
➔ Situations involving additive structures

Uniting

(DETERMINE THE SET)

Thomas bought a box of pencils, a backpack and a book at the flea market.

\$2.65



\$3.45



\$6.80



I assign situations with several items of given information.



?

pencils	2			25	25		10	5
backpack	2	2	2	25	25	25		5
book	2	1		25			10	10

\$2.65

\$6.80

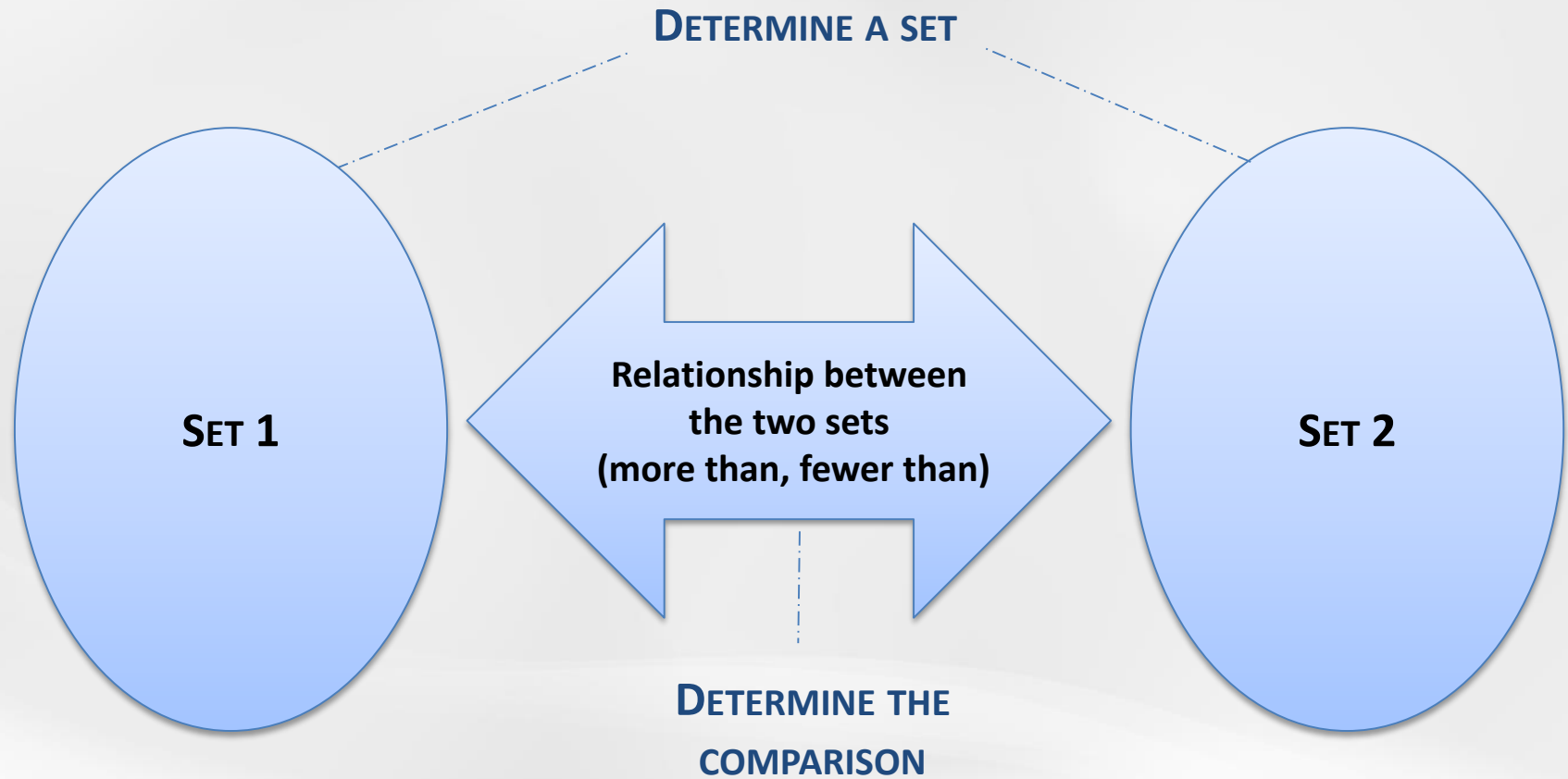
\$3.45

How much did Thomas spend?

➔ Situations involving additive structures

Cycles 1, 2 and 3

Comparing

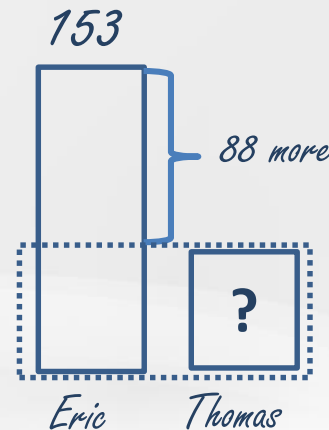
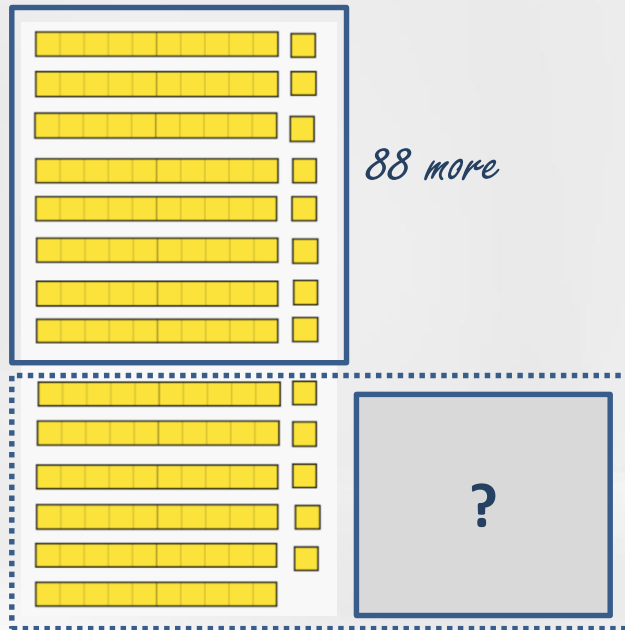


➔ Situations involving additive structures

Comparing

(DETERMINE ONE OF THE SETS)

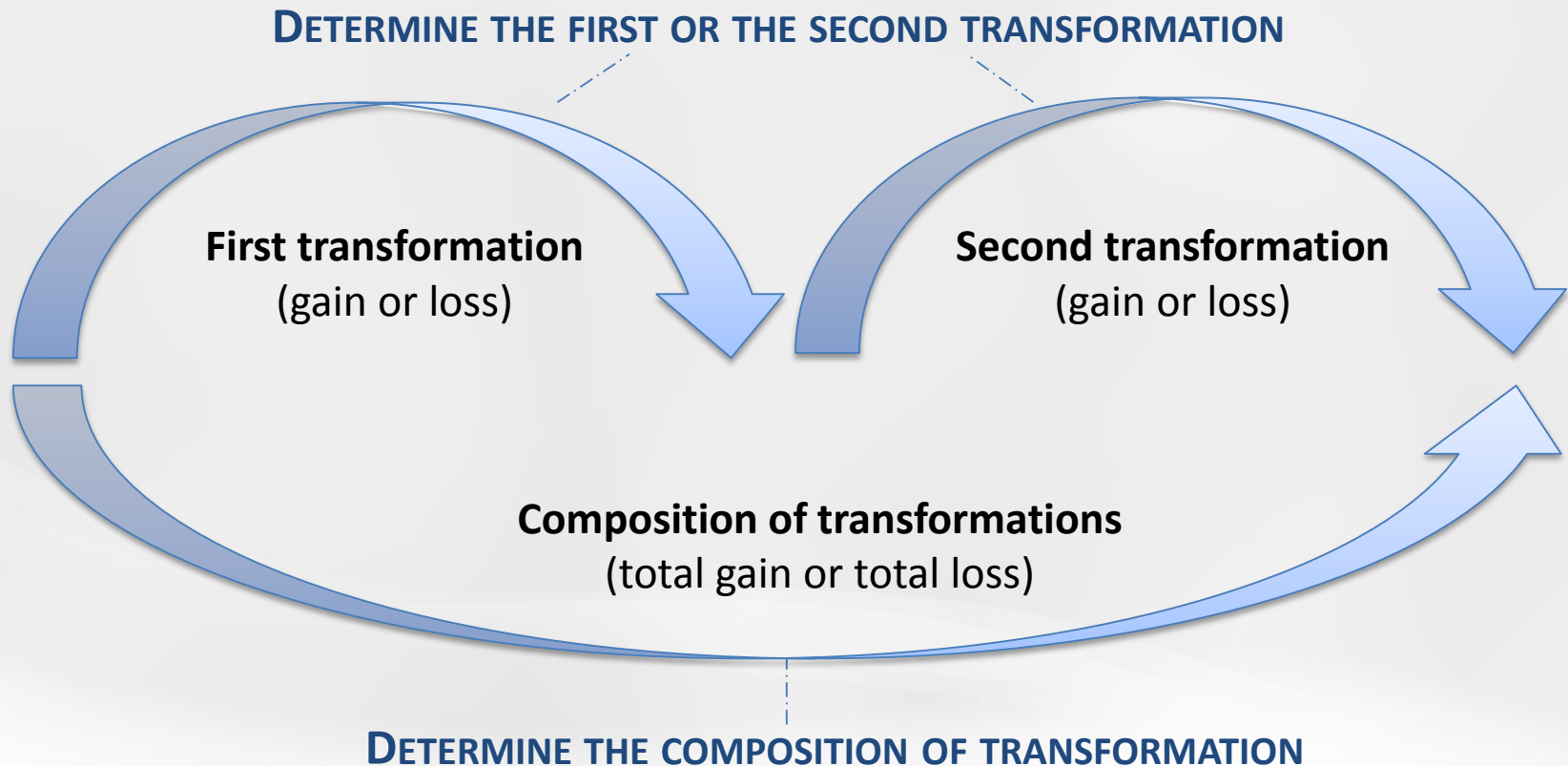
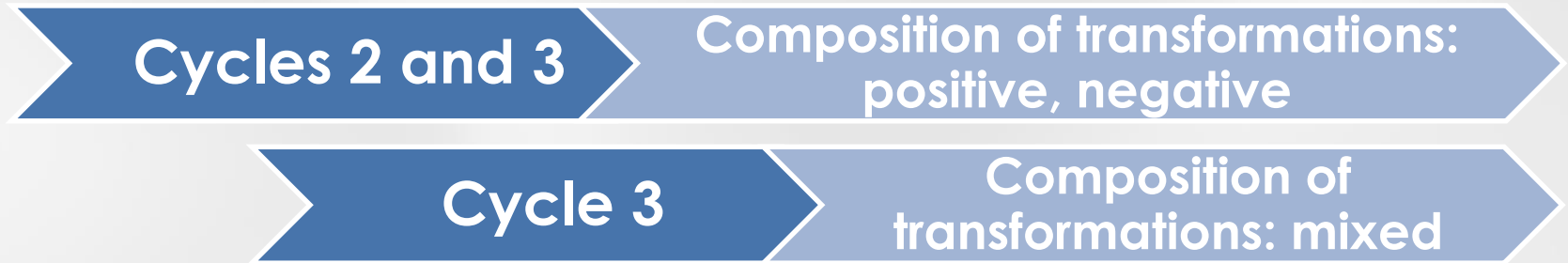
Eric has 153 figurines. He has 88 more than Thomas.
How many figurines does Thomas have?



$$153 = \square + 88$$

$$153 - 88 = \square$$

➔ Situations involving additive structures

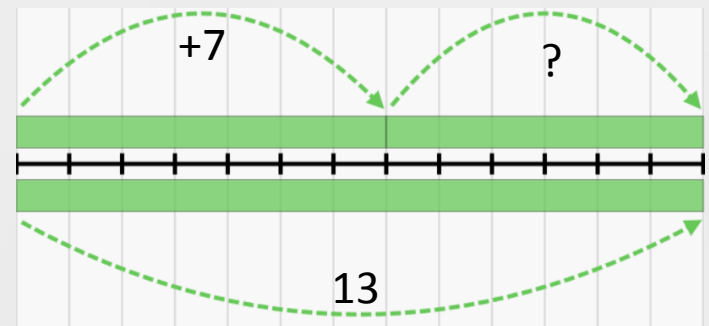
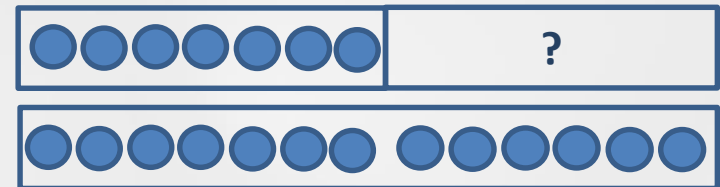


➔ Situations involving additive structures

Composition of transformations (positive)

(DETERMINE THE TRANSFORMATION INVOLVED)

Yesterday, Gus received 7 objects. Today he has received more, but we do not know how many.



Given that he has received 13 objects in the past 2 days, how many objects did he receive today?

$$7 + \square = 13$$

$$13 - 7 = \square$$

Uses objects, diagrams or equations to represent a situation and conversely, describes a situation represented by objects, diagrams or equations (use of different meanings of addition and subtraction)

Meaning of operations involving numbers						
<div>→</div> Student constructs knowledge with teacher guidance.	Elementary					
	Cycle One		Cycle Two		Cycle Three	
	1	2	3	4	5	6
<div>★</div> Student applies knowledge by the end of the school year.						
<div></div> Student reinvests knowledge.						
A. Natural numbers less than . . .	1000		100 000		1 000 000	
1. Determines the operation(s) to perform in a given situation	→	★	→	★	→	★
2. Uses objects, diagrams or equations to represent a situation and conversely, describes a situation represented by objects, diagrams or equations (use of different meanings of addition and subtraction)						
a. transformation (adding, taking away), uniting, comparing	→	★	→	★	→	★
b. composition of transformations: positive, negative			→	★	→	★
c. composition of mixed transformations					→	★

Uses objects, diagrams or equations to represent a situation and conversely, describes a situation represented by objects, diagrams or equations (use of different meanings of addition and subtraction)

B. Decimals up to . . .	1	2	3	4	5	6
	hundredths			thousandths		
1. Uses objects, diagrams or equations to represent a situation and conversely, describes a situation represented by objects, diagrams or equations (use of different meanings of addition and subtraction)						
a. transformation (adding, taking away), uniting, comparing			→	★	→	★
b. composition of transformations: positive, negative			→	★	→	★
c. composition of mixed transformations					→	★

C. Fractions	1	2	3	4	5	6
1. Uses objects, diagrams or equations to represent a situation and conversely, describes a situation represented by objects, diagrams or equations (use of different meanings of addition, subtraction and multiplication by a natural number)					→	★

Situations involving multiplicative structures



Rectangular array

Repeated addition

Cartesian product

Sharing

Number of times x goes into y

Situations involving multiplicative structures

Cycle 2

Cycle 3

Area

Volume

Repeated subtraction

Comparing

➔ Situations involving multiplicative structures

Cartesian product

Gus has 4 shirts and 3 pairs of pants.
How many different outfits can he wear?



	C1	C2	C3	C4
P1	P1C1	P1C2	P1C3	P1C4
P2	P2C1	P2C2	P2C3	P2C4
P3	P3C1	P3C2	P3C3	P3C4

$$3 \times 4 = \square$$

$$4 \times 3 = \square$$

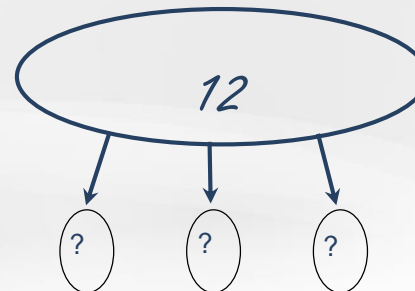
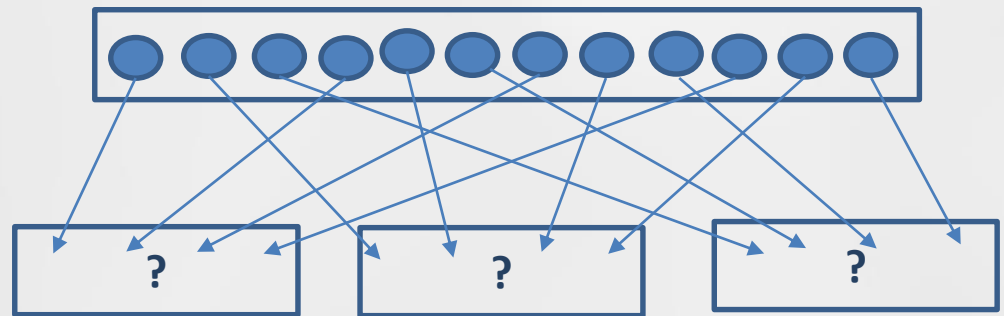
<http://illuminations.nctm.org/Activity.aspx?id=3540>

➔ Situations involving multiplicative structures

Sharing

There are 12 pencils to be distributed equally among 3 friends.

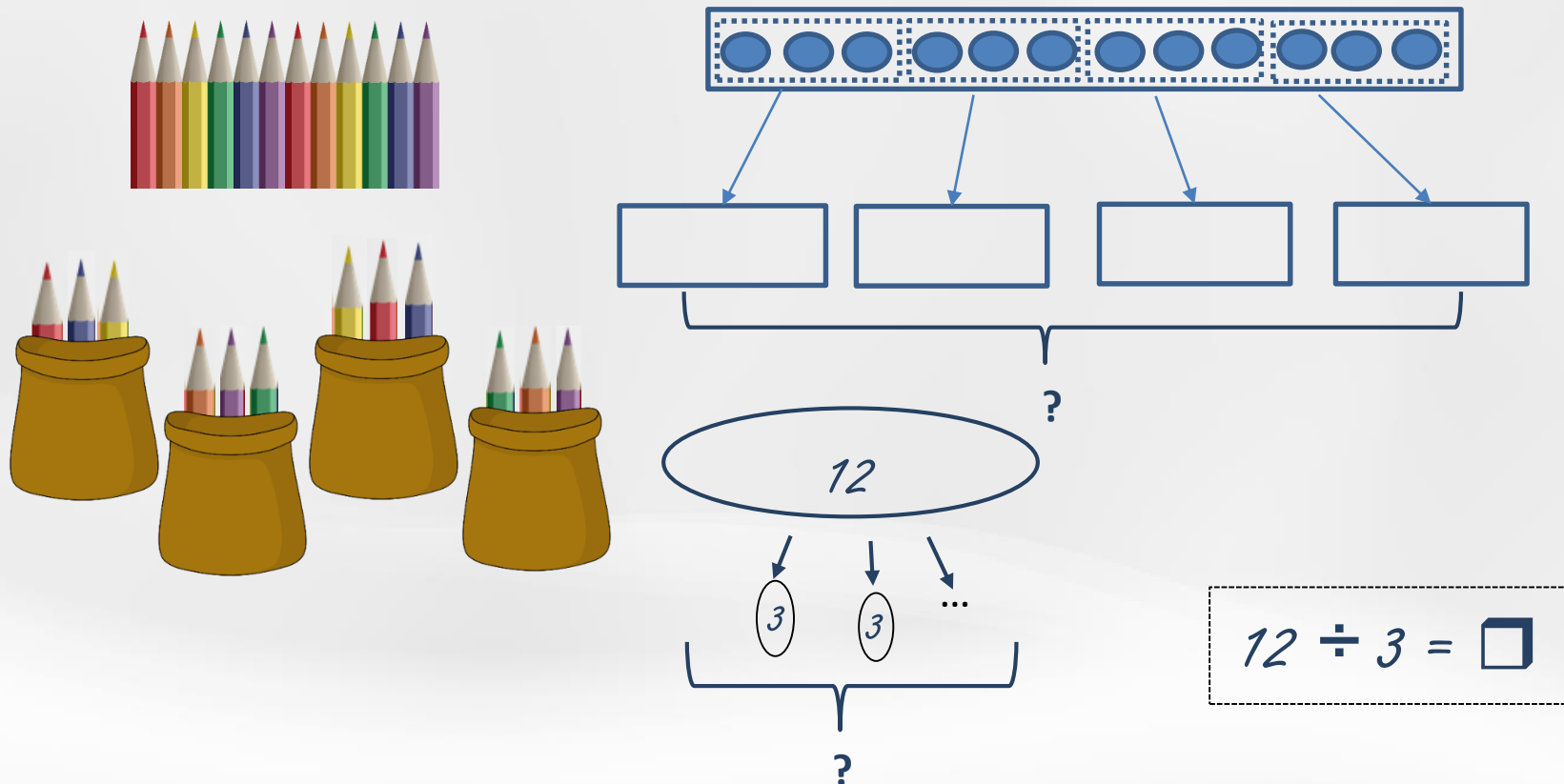
How many pencils will each friend have?



$$12 \div 3 = \square$$

Number of times x goes into y

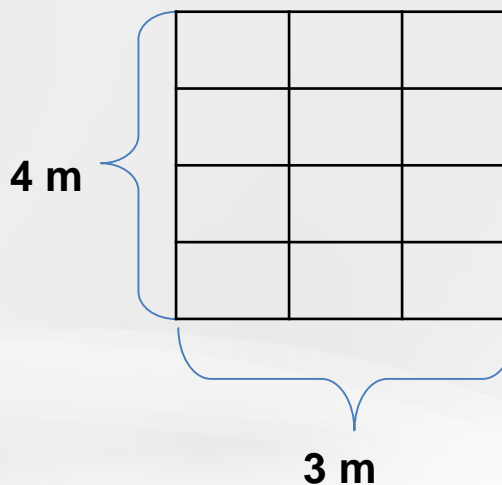
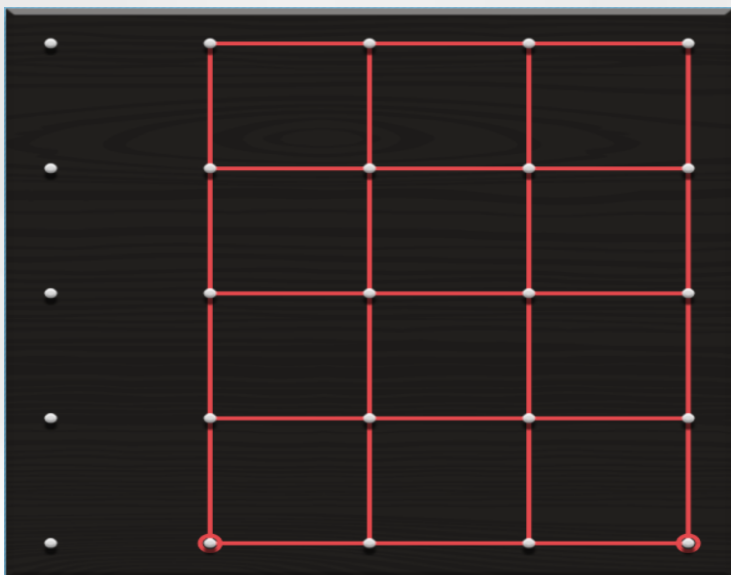
You want to put 12 pencils in some bags. Each bag can hold 3 pencils. How many bags will you need?



➔ Situations involving multiplicative structures

Area

A flower bed containing 35 daisies is 3 m (or unit squares) wide by 4 m (or unit squares) long. What is the area of this flower bed?



I assign situations that include superfluous data.



$$4 \times 3 = \square$$

$$3 \times 4 = \square$$

Comparing

Gus has 3 objects. Melanie has 4 times as many objects.
How many objects does Melanie have?



Gus

Melanie

$$4 \times 3 = \square$$

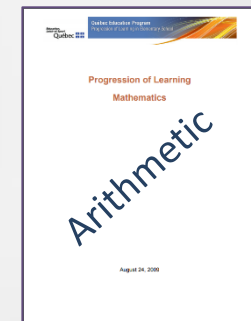
$$3 \times 4 = \square$$



Uses objects, diagrams or equations to represent a situation and conversely, describes a situation represented by objects, diagrams or equations (use of different meanings of multiplication and division)

Meaning of operations involving numbers						
<div>→</div> Student constructs knowledge with teacher guidance.	Elementary					
	Cycle One		Cycle Two		Cycle Three	
	1	2	3	4	5	6
★ Student applies knowledge by the end of the school year.						
Student reinvests knowledge.						
A. Natural numbers less than . . .						
	1000		100 000		1 000 000	
3. Uses objects, diagrams or equations to represent a situation and conversely, describes a situation represented by objects, diagrams or equations (use of different meanings of multiplication and division)						
a. rectangular arrays, repeated addition, Cartesian product, sharing, and number of times x goes into y (using objects and diagrams)	→	★				
b. rectangular arrays, repeated addition, Cartesian product, area, volume, repeated subtraction, sharing, number of times x goes into y , and comparisons (using objects, diagrams or equations)			→	★	→	★
B. Decimals up to . . .						
	1	2	3	4	5	6
	hundredths			thousandths		
2. Uses objects, diagrams or equations to represent a situation and conversely, describes a situation represented by objects, diagrams or equations (use of different meanings of multiplication and division: rectangular arrays, Cartesian product, area, volume, sharing, number of times x goes into y , and comparisons)			→	★	→	★
C. Fractions						
	1	2	3	4	5	6
1. Uses objects, diagrams or equations to represent a situation and conversely, describes a situation represented by objects, diagrams or equations (use of different meanings of addition, subtraction and multiplication by a natural number)					→	★

Progression of Learning Arithmetic



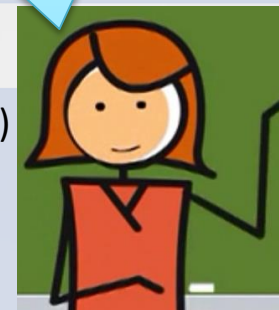
Understanding and writing numbers

Natural numbers less than . . .
Fractions (using objects or drawings)
Decimals up to . . .
Integers

Meaning of operations

Natural numbers less than . . .
Decimals up to . . .
Fractions

I make sure to
look at the
benchmarks
indicated in
the section on
operations.



Operations

Natural numbers (based on the benchmarks for each cycle)
Fractions (using objects or drawings)
Decimals
Using numbers

Develops processes for written computation (addition and subtraction)

A. Natural numbers (based on the benchmarks for each cycle)	1	2	3	4	5	6
4. Develops processes for written computation (addition and subtraction)						
a. Uses his/her own processes as well as objects and drawings to determine the sum or difference of two natural numbers less than 1000	→	★				
b. Uses conventional processes to determine the sum of two natural numbers of up to four digits			→	★		
c. Uses conventional processes to determine the difference between two natural numbers of up to four digits whose result is greater than 0			→	★		
B. Fractions (using objects or diagrams)	1	2	3	4	5	6
3. Adds and subtracts fractions when the denominator of one fraction is a multiple of the other fraction(s)				→	★	
C. Decimals	1	2	3	4	5	6
3. Develops processes for written computation						
a. adds and subtracts decimals whose result does not go beyond the second decimal place			→	★		

Develops processes for written computation (multiplication and division)

A. Natural numbers (based on the benchmarks for each cycle)	1	2	3	4	5	6
7. Develops processes for written computation (multiplication and division)						
a. Uses his/her own processes as well as materials and drawings to determine the product or quotient of a three-digit natural number and a one-digit natural number, expresses the remainder of a division as a fraction, depending on the context			→	★		
b. Uses conventional processes to determine the product of a three-digit natural number and a two-digit natural number					→	★
c. Uses conventional processes to determine the quotient of a four-digit natural number and a two-digit natural number, expresses the remainder of a division as a decimal that does not go beyond the second decimal place					→	★
B. Fractions (using objects or diagrams)	1	2	3	4	5	6
4. Multiplies a natural number by a fraction					→	★
C. Decimals	1	2	3	4	5	6
3. Develops processes for written computation						
b. multiplies decimals whose product does not go beyond the second decimal place					→	★
b. divides a decimal by a natural number less than 11					→	★

Determines the missing term in an equation (relationships between operations)

A. Natural numbers (based on the benchmarks for each cycle)	1	2	3	4	5	6
5. Determines the missing term in an equation (relationships between operations): $a + b = \square$, $a + \square = c$, $\square + b = c$, $a - b = \square$, $a - \square = c$, $\square - b = c$	→	★				
8. Determines the missing term in an equation (relationships between operations): $a \times b = \square$, $a \times \square = c$, $\square \times b = c$, $a \div b = \square$, $a \div \square = c$, $\square \div b = c$			→	→	→	★

Elementary-Level Mathematics

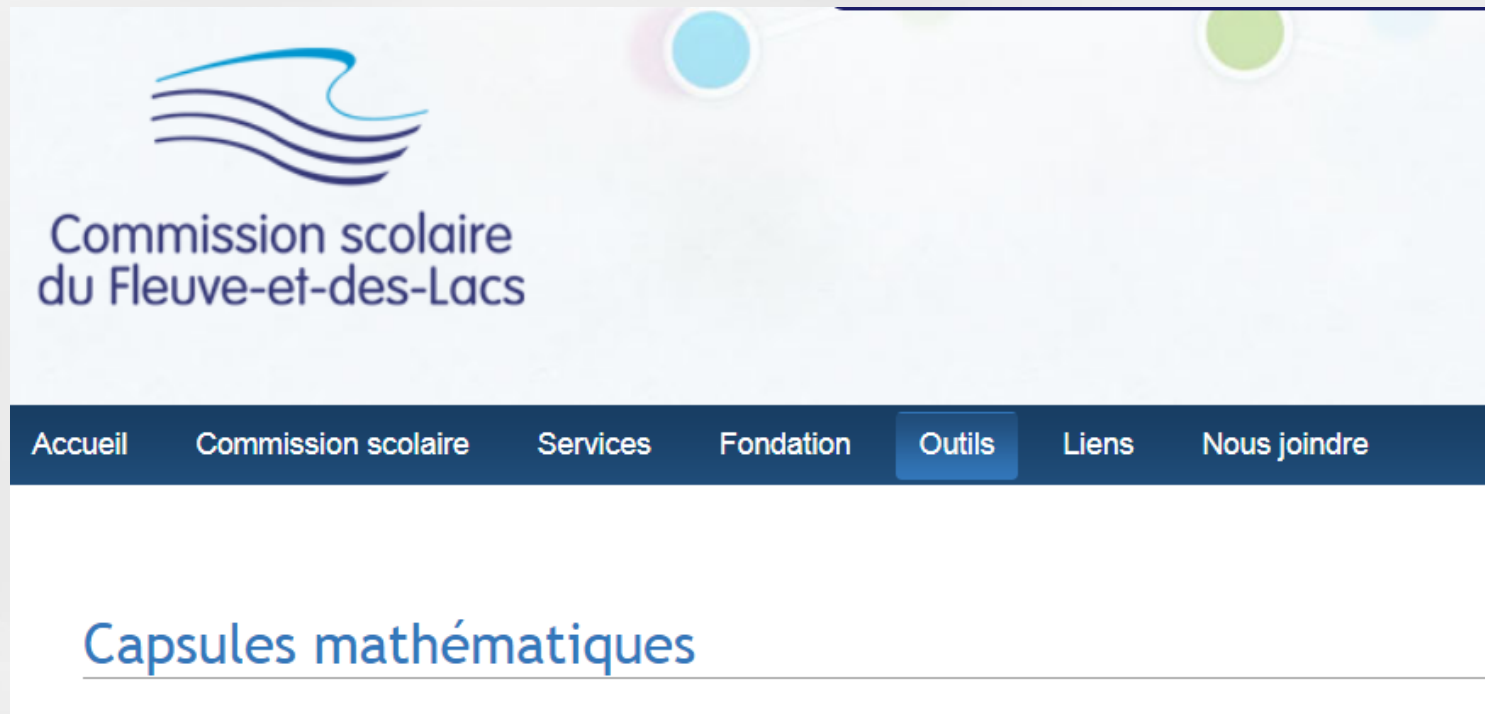
Use of different meanings of addition, subtraction, multiplication and division

Table of Contents

Introduction.....	4
Situations involving additive structures	5
Transformation	6
Uniting	7
Comparing.....	7
Composition of transformations: positive, negative.....	8
Composition of mixed transformations	9
Situations involving multiplicative structures	10
Rectangular array.....	11
Repeated addition	11
Cartesian product.....	11
Sharing	11
Number of times x goes into y	11
Area	12
Volume	12
Repeated subtraction.....	12
Bibliography and Webography	13

For more information on the different situations, please see the companion document

Commission scolaire du Fleuve-et-des-Lacs



[www.csfl.qc.ca/index.php/outils/outils-pedagogiques/capsules-mathématiques](http://www.csfl.qc.ca/index.php/outils/outils-pedagogiques/capsules-mathematiques)

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