Optimizing the Development of the Mathematical Competencies to Help Students Make Sense of the Mathematics

Elementary

Québec

👍 Welcome!

➤ **Raymond Nolin**, Elementary school mathematics teacher (loan of service)

➤ **Esther Veilleux**, Secondary school mathematics teacher (loan of service)

➤ **Geneviève Dupré**, Coordinator, Mathematics program

Objectives of the meeting

- Provide a clear explanation of intradisciplinary links (between competencies and between branches of mathematics) and interdisciplinary links.

- Explain how the development of the competencies and mathematical learning can be optimized.

- Provide examples of learning activities that foster the development of the competencies and make learning more meaningful.

Presentation outline

1. Connections between the competencies
2. Connections between the branches of mathematics
3. Interdisciplinary links
4. Teaching strategies
5. Reflection questions

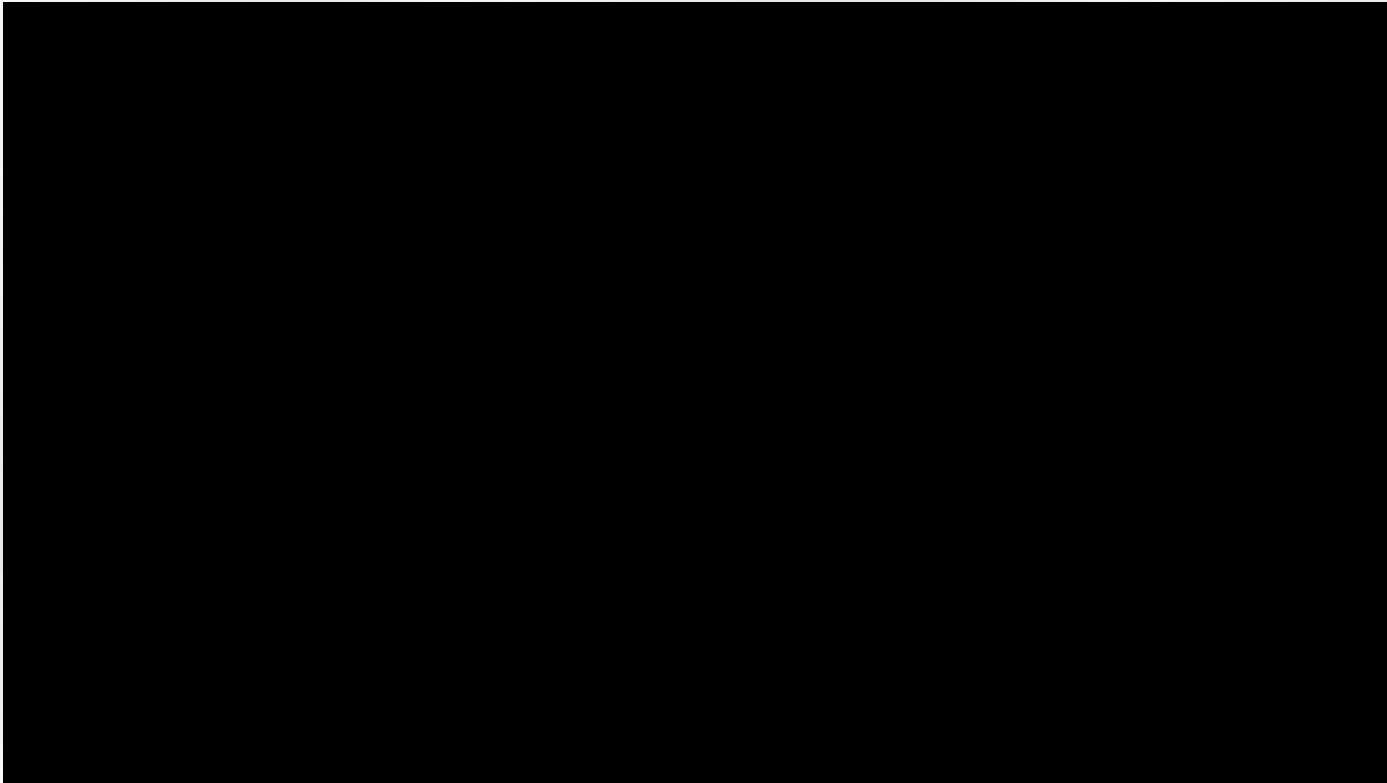# To foster reflection...

Students do not learn simply by being told.

They must experience cognitive dissonance,
which is central to learning mathematics.

– Picard, 2018

[free translation]
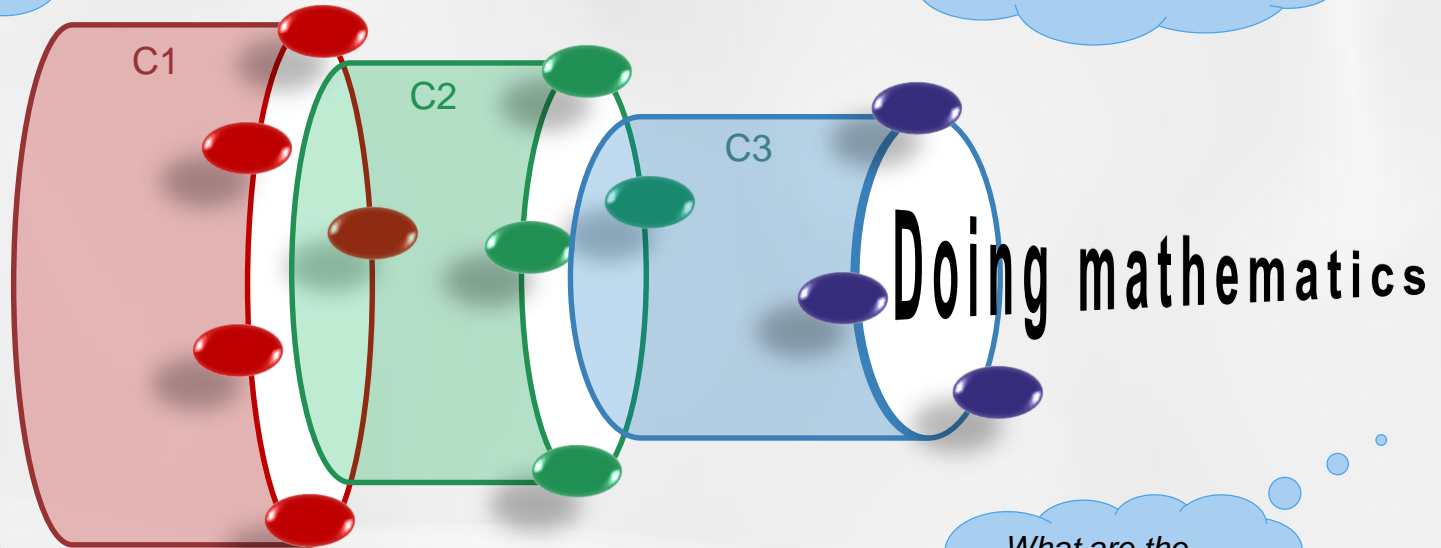
Video on mathematical competencies
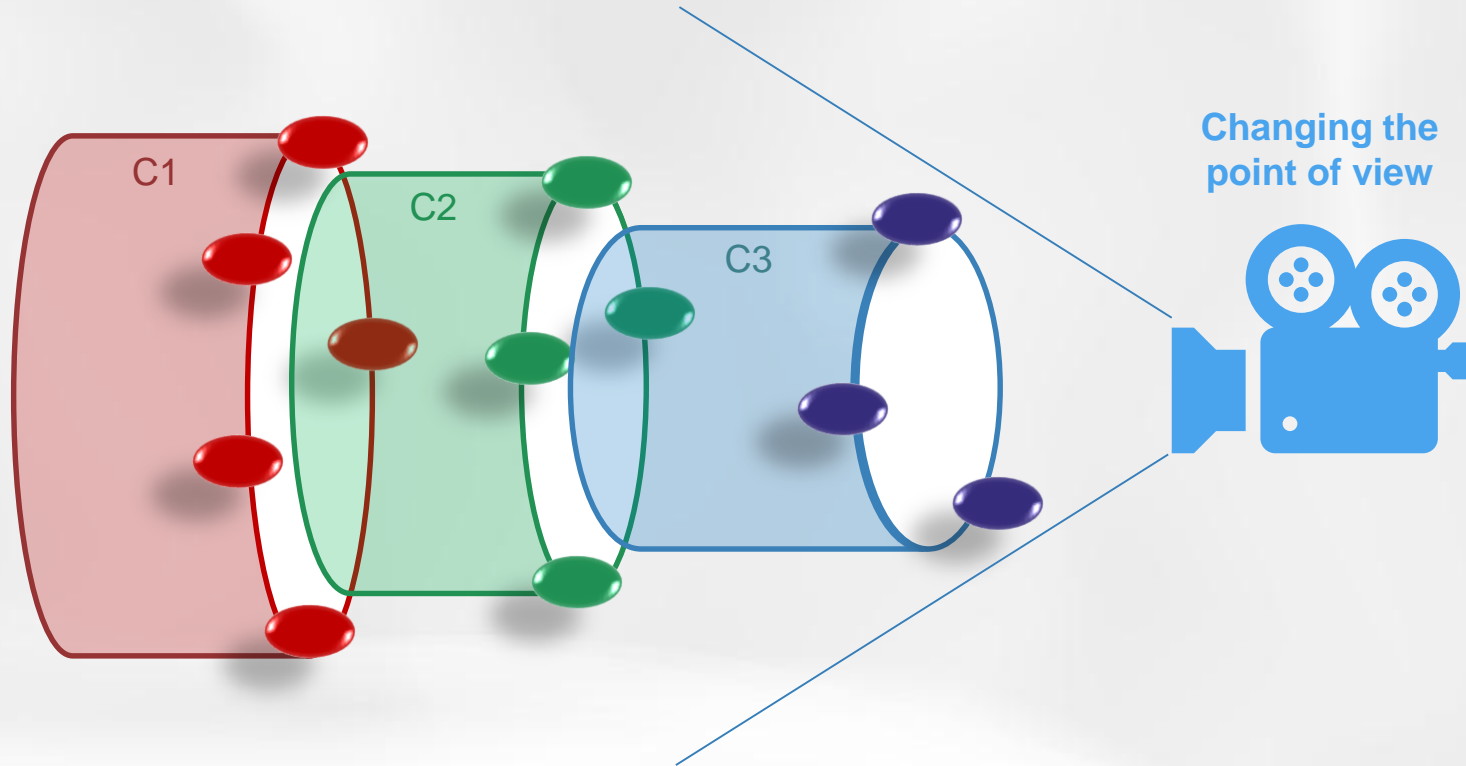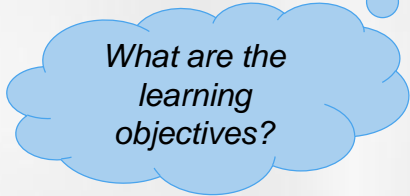
1 Connections between the competencies
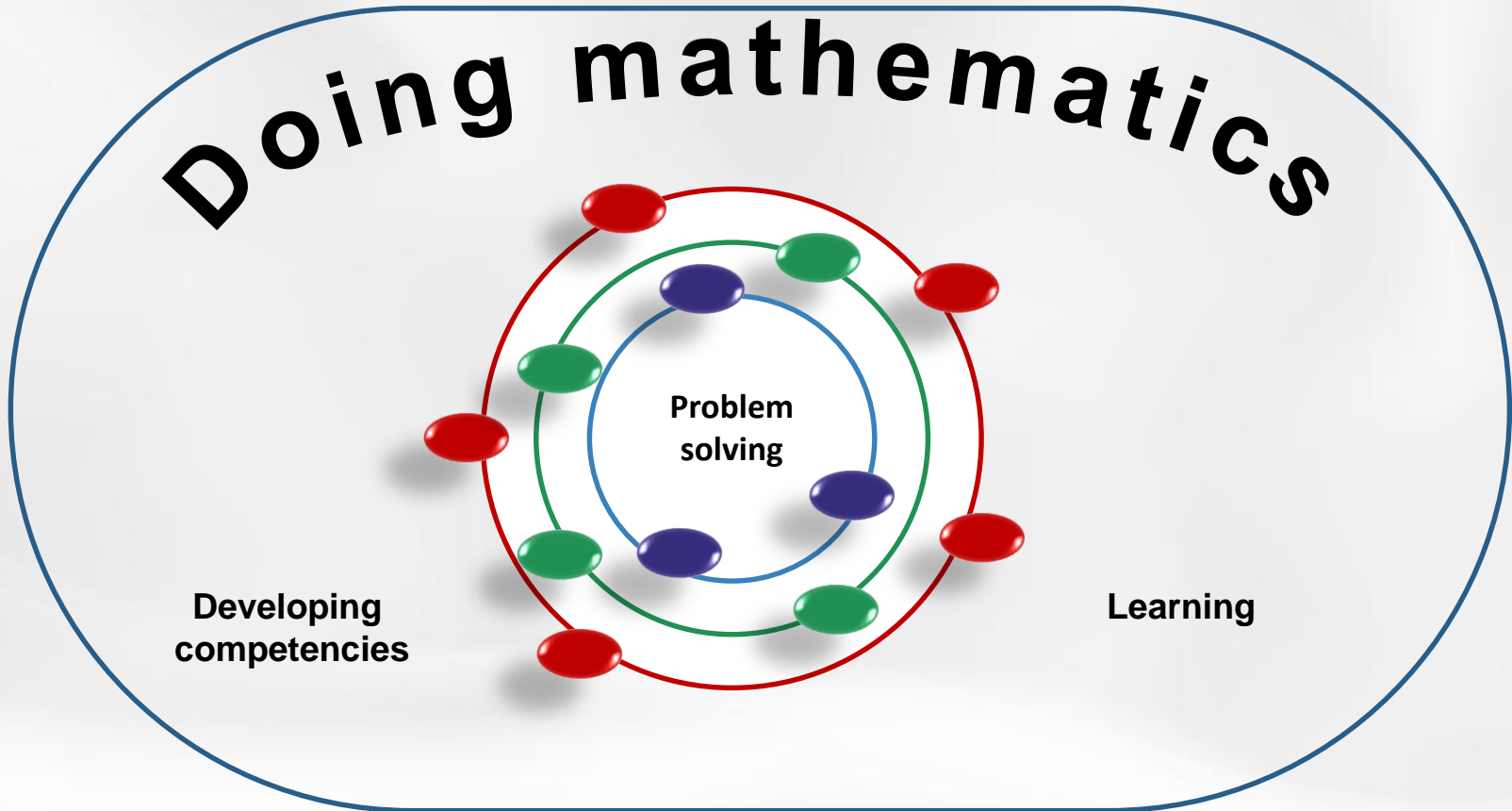
Learning objectives

- To learn to solve mathematical problems.
- To use various problem–solving strategies.
- To use different types of representations.
- To introduce students to a new concept.

- To apply a variety of concepts and processes within the same problem
- To have students use mathematical language to explain their procedure and solution(s).

Example of a learning activity

- Objective of the activity:

  To illustrate how problem solving helps to develop the three mathematical competencies.

- Activity format: 5–5–10

Individual work (5 minutes) → Small-group discussion (5 minutes) → Whole-group discussion (10 minutes)
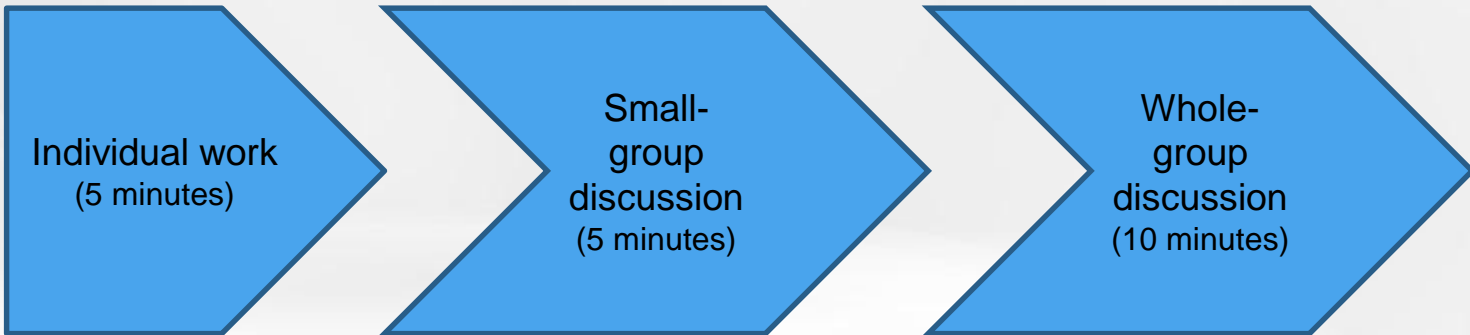
Example of a learning activity

- **<u>Ordering chicken nuggets for a party</u>**

You are planning a party and would like to order chicken nuggets from your favourite restaurant. The restaurant sells the nuggets in packages of 6, 9 or 20. However, you are not sure how many nuggets to order. Using the package sizes for 6, 9 and 20 nuggets offered by the restaurant, determine any number of chicken nuggets from 1 to 100 that you can order.

Adapted from the task *Ordering for the Holiday Party* by Roy, Allen and Thacker (2020).

Whole-group discussion

- Which elementary cycle(s) is this learning activity best suited for?

Learning objectives

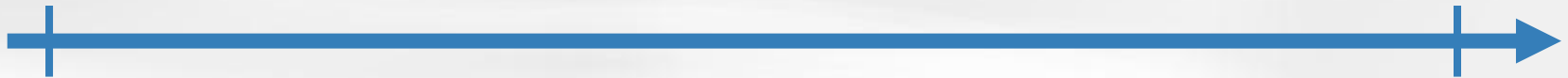- What could be the learning objectives for this type of activity?

Beginning of learning process   End of learning process

Connections between the competencies

Learning objective

Arithmetic

Probability

Geometry

To solve a situational problem related to mathematics

Construction of identity

Student

World view   Empowerment

To reason using mathematical concepts and processes

To communicate by using mathematical language

Planning strategies

Comprehension strategies

Affective strategies

Organization strategies

Communication strategies

Development strategies

Resource management strategies

Retention strategies

Statistics

Measurement

Regulation strategies

Generalization strategies

For examples of cognitive and metacognitive strategies, consult the Progression of Learning – Mathematics – Elementary, 2009, pp. 23-24.

Encouraging reflection…

*"The best we can do for students
is to have them ask the right questions."*

– Cantor, 2002, cited in OME, 2004

For more information about avenues for discussion, see the tables in each section of the document [New Approaches, New Solutions in Mathematics. Fostering Student Success in Disadvantaged Areas](#).

2 **Connections between the branches of mathematics**

- The decompartmentalization of the branches of mathematics:
  - ○ Helps students develop a greater understanding of concepts as well as greater flexibility and fluency
  - ○ Makes the mathematics more meaningful
  - ○ Helps students develop mathematical thinking as a whole
  - ○ Optimizes learning by helping students make connections between mathematical concepts and processes.
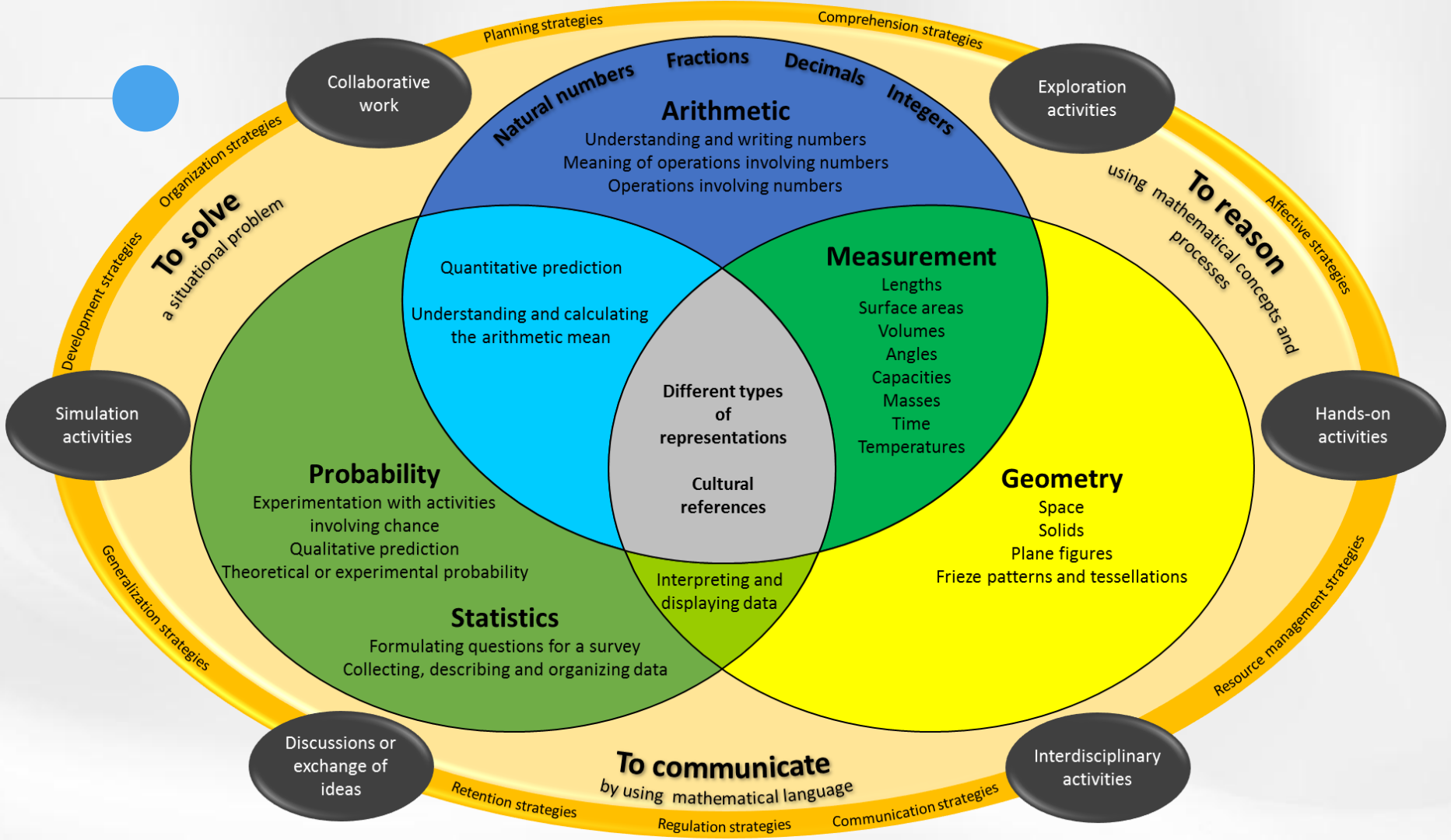
Arithmetic

Understanding and writing numbers

Natural numbers	Fractions	Decimals	Integers

Reads and writes numbers

Compares numbers

Arranges numbers in increasing or decreasing order

Represents numbers in a variety of ways

Locates numbers using different visual aids (number line)

Identifies equivalent expressions

Counts or recites counting rhymes involving numbers	Matches a decimal or percentage to a fraction	
Counts collections (using objects or drawings)	Identifies fractions related to everyday items	Understands the role of the decimal point
Composes and decomposes a number	Matches a fraction to part of a whole	Composes and decomposes a number
Describes patterns	Identifies different meanings	Approximates
Identifies properties of numbers	Distinguishes a numerator from a denominator	
Classifies numbers based on their properties		
Approximates a collection, using objects or drawings		
Represents the power of a number		
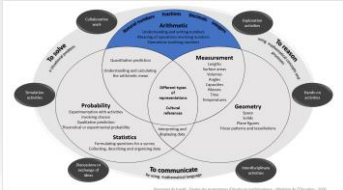
Geometry

Space
- Gets his/her bearings and locates objects
- Locates objects in a plane
- Locates objects on an axis
- Locates points in a Cartesian plane

Solids
- Compares objects or parts of objects in the environment with solids
- Compares and constructs solids
- Identifies the main solids
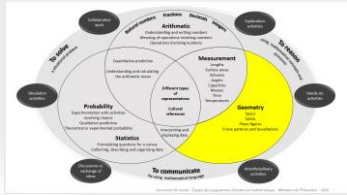- Represents the different faces of a prism or pyramid
- Describes prisms and pyramids in terms of faces, vertices and edges
- Classifies prisms and pyramids
- Matches the net of a prism to the corresponding prism
- Matches the net of a pyramid to the corresponding pyramid
- Matches the net of a convex polyhedron to the corresponding convex polyhedron
- Tests Euler's theorem

Plane figures
- Compares and constructs figures
- Identifies plane figures
- Describes plane figures
- Describes convex and nonconvex polygons
- Identifies and constructs parallel lines and perpendicular lines
- Describes and classifies quadrilaterals
- Describes and classifies triangles
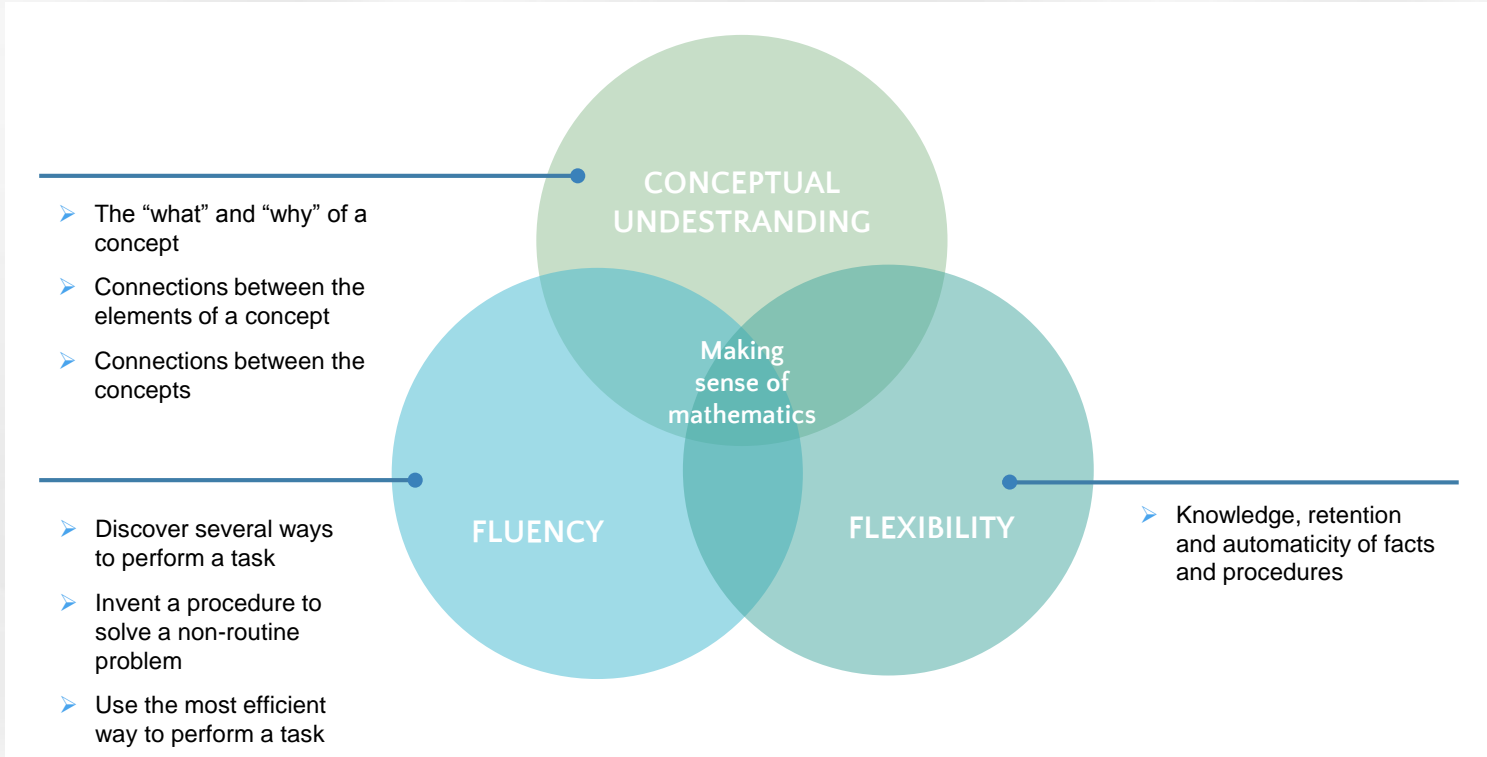- Describes circles

Frieze patterns and tessellations
- Identifies congruent figures
- Observes and produces patterns using geometric figures
- Observes and produces frieze patterns and tessellations

Foster flexibility and fluency in students

> The "what" and "why" of a concept

> Connections between the elements of a concept

> Connections between the concepts

> Discover several ways to perform a task

> Invent a procedure to solve a non-routine problem

> Use the most efficient way to perform a task

CONCEPTUAL UNDESTRANDING

Making sense of mathematics

FLUENCY

FLEXIBILITY

> Knowledge, retention and automaticity of facts and procedures

Source: Référentiel d'intervention en mathématique, (MEES, 2019) (English version forthcoming)

Connections between the branches of mathematics

- A problem's context can sometimes help students discover, reapply or deepen their knowledge of elements associated with various branches of mathematics.

  - Certain **probability** tasks help students reapply their knowledge of fractions, decimals and percentages.

  - Certain **measurement** tasks help students construct the meaning of natural numbers and decimals.

  - Certain **geometry** tasks help students develop their understanding of the concept of fractions.

Example of a learning activity

- **<u>The disk game</u>**

A bag contains circular disks. In the bag, there are 5 blue disks, 6 red disks, 3 green disks and 2 yellow disks. Several orange disks are added to the bag. All the disks in the bag are identical except for colour.

A disk is now randomly selected from the bag. The probability of a blue or green disk being selected is now $\frac{2}{5}$.

How many orange disks were added to the bag?

Example of a learning activity

- Adapting the tasks to students' abilities

A bag contains circular disks. In the bag, there are 5 blue disks, 6 red disks, 3 green disks and 2 yellow disks. Several orange disks are added to the bag. All the disks in the bag are identical except for colour.

Which of the following events are certain, possible or impossible?

Select a red disk.   **Select a disk that is not black.**

Select a purple disk.   **Select a yellow disk.**

Select a green disk.   **Select a black disk.**

Adapted from the task *Playing With Disks* from the Centre for Education in Mathematics and Computing at the University of Waterloo (2020).

Example of a learning activity

- Adapting the tasks to students' abilities

A bag contains circular disks. In the bag, there are 5 blue disks, 6 red disks, 3 green disks and 2 yellow disks. Several orange disks are added to the bag. In total, there are 28 identical disks in the bag.

Which of the following events is most likely to occur?

Select a red disk.	**Select an orange disk.**
Select a purple disk.	**Select a yellow disk.**
Select a green disk.	**Select a blue disk.**

Connections between the branches of mathematics

- An **ecological approach** should be considered by teachers who wish to emphasize connections between the branches of mathematics.

- For example, it may be useful to use measurement activities when teaching students about decimals, and vice versa.

- A better understanding of the competencies and strategies acquired by students helps teachers choose learning activities that clearly emphasize connections between the branches of mathematics.
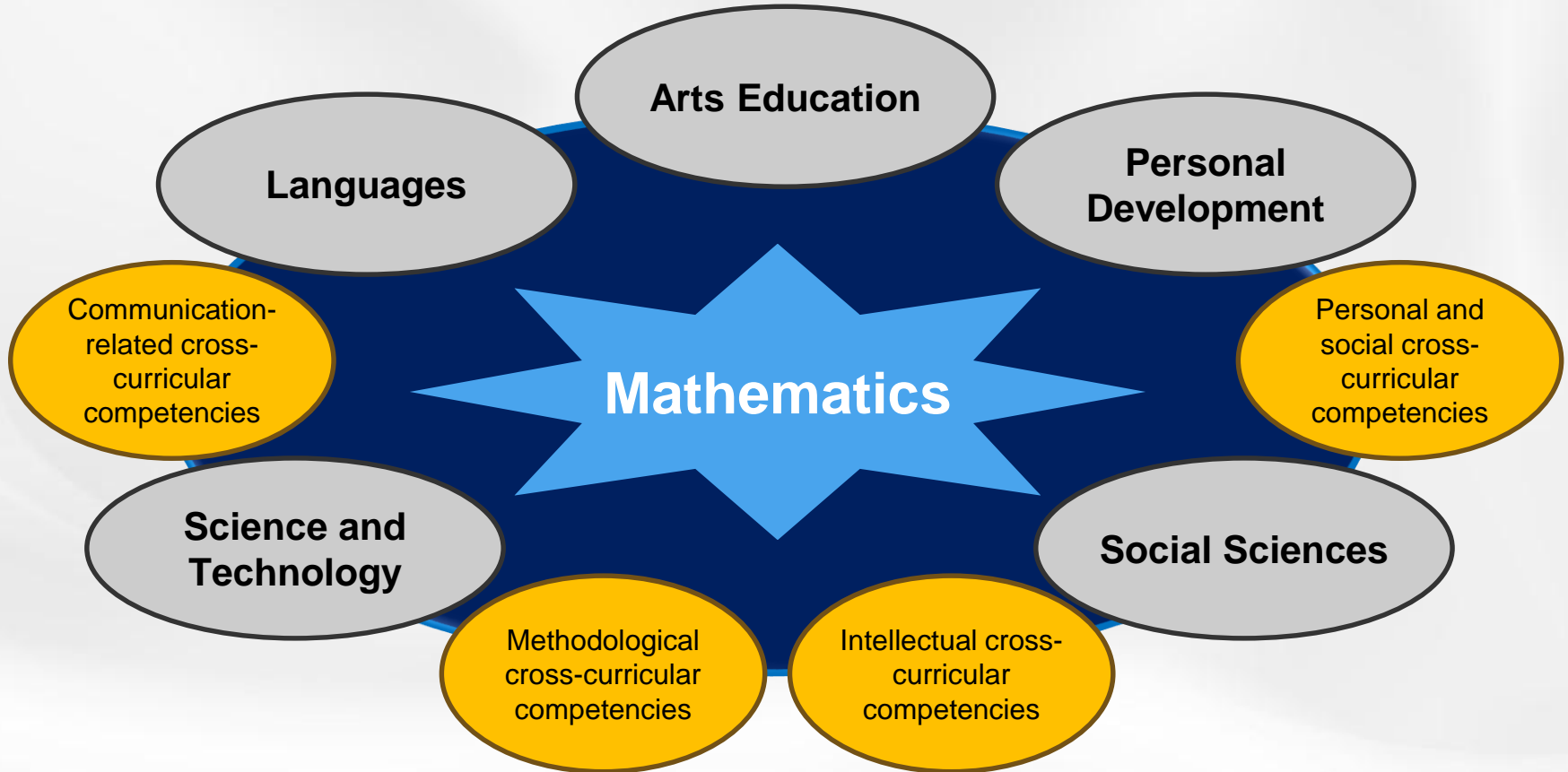
To foster reflection …

Students can come to realize that they are capable of making sense of mathematics.

To teach effectively means to engage students at their level, thus enabling them to come up with or assimilate new ideas that will help them make sense of mathematics.

– Van de Walle & Lovin, 2018

Interdisciplinary links

Using mathematical competencies in Social Sciences

- *Gets his/her bearings in space* and *Location in space*

Gets his/her bearings and locates objects in space

Orients himself/herself in space, a simple drawing, an illustration or a scale model

- *Locates numbers* and *Orients himself/herself in time*

Locates natural numbers using different visual aids, particularly a number line

Situates on a graduated time line events and people related to the history of the society

Example of a learning activity

- <u>**Time line**</u>

In Elementary Cycle Two, students explore various societies at different periods in history. To help students orient themselves in time, it may be interesting to suggest that they create a time line.

The following events and periods could be placed on the time line.

Columbus's discovery of America (1492)

The founding of Québec City (1608), Trois-Rivières (1634) and Montréal (1642)

The Great Peace of Montréal (1701)

The intendancy of Gilles Hocquart (1729-1748)

Example of a learning activity

- **<u>Games for everyone</u>**

During the winter Olympics, several competitions are held during the day while the students are in school. Mr. Paco would like his students to take a few short breaks so they can watch the competitions. First, he has to find out which competitions his students like the most. Help Mr. Paco come up with survey questions he can ask his students to find out which competitions they like the most.

"Do you like downhill skiing?"   **"Do you prefer speed skating or hockey?"**

"What is your favourite competition?"

Other examples

- Mathematics can be found in *Ethics and Religious Culture* content.

- Mathematics can be found in *Physical Education and Health.*

- Mathematics can be found in the *Visual Arts.*

- Mathematics can be found in *Music.*

- Several of these interdisciplinary links can also be used to explore various **cultural references**.
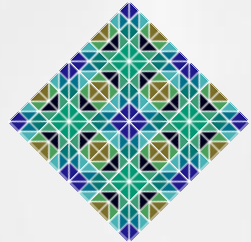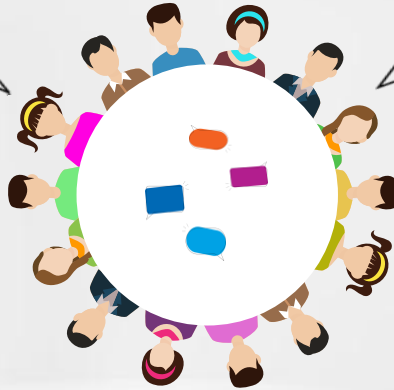
To sum up...
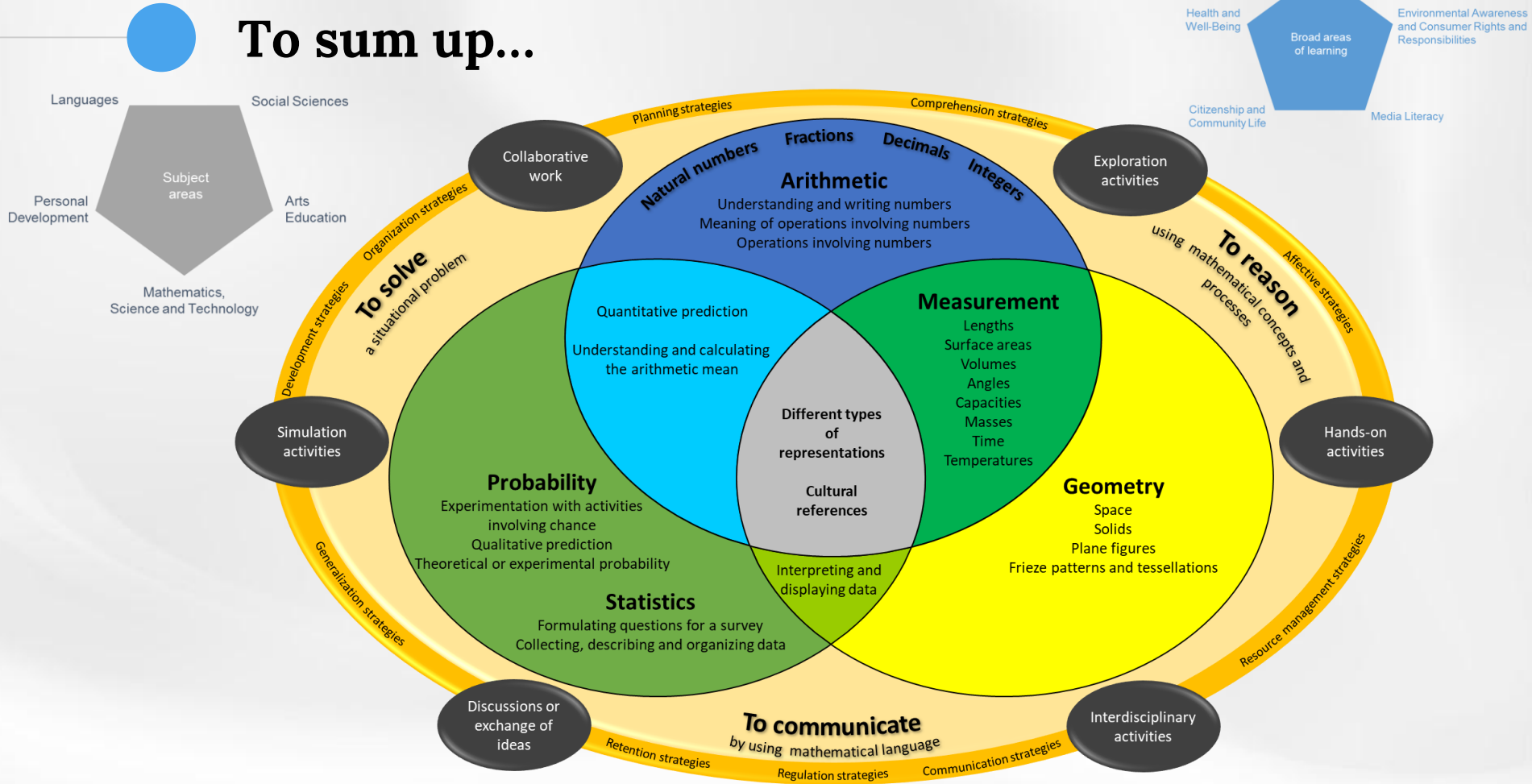
4 Teaching strategies that foster the optimization of learning

Problem solving

Mathematical discussions and routines

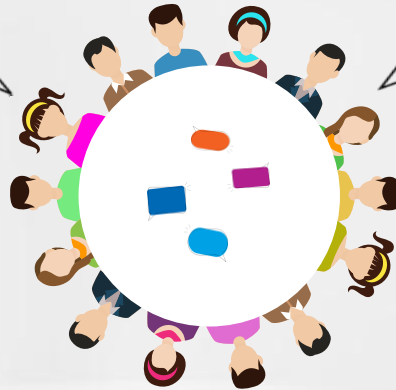Classroom climate

Various types of reasoning

Teaching strategies that foster the optimization of learning

Engaging and concrete activities

Concrete examples based on everyday life

Error as a learning opportunity

A variety of resources

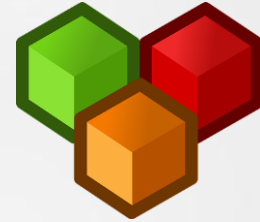# Teaching strategies that foster the optimization of learning

- **Using a variety of resources:**

Authentic material
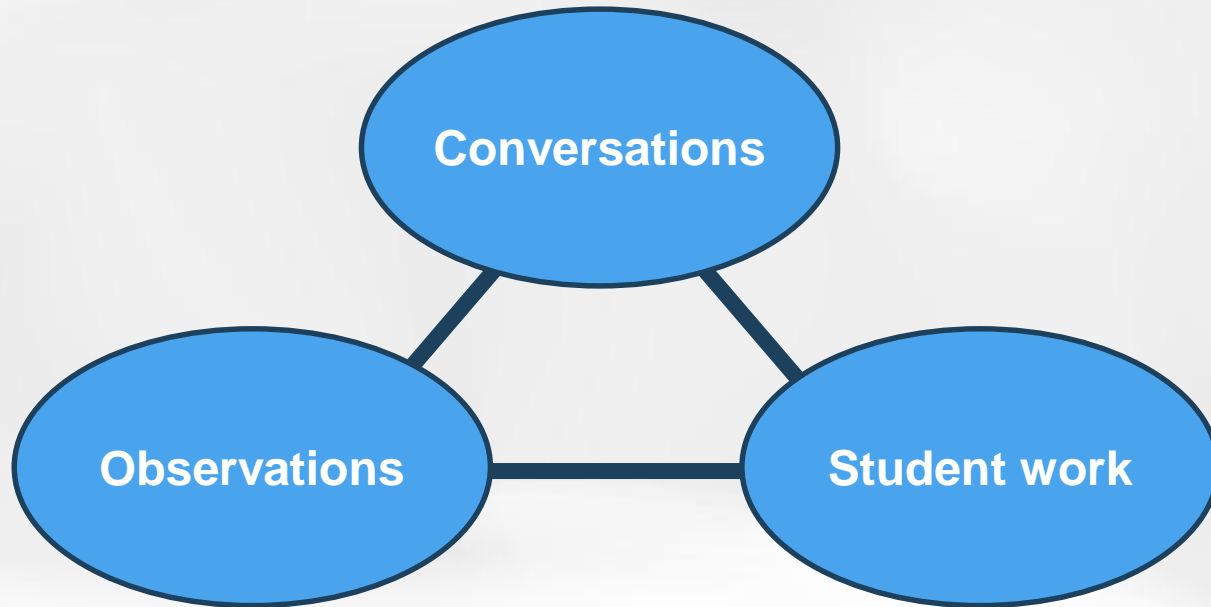

Ecological approach

Manipulatives

ICT and programming

Teaching strategies that foster the optimization of learning

- Providing varied feedback:

Teaching strategies that foster the optimization of learning

Ask questions

Plan

Guide

Provide details

Learning objective

Diversify

Food for thought…

No single approach can guarantee success for all students. Teachers' practices must comprise a balanced mixture of different approaches, teaching strategies and organizational arrangements.

– UNESCO, 2000, from *New Approaches, New Solutions*

Reflection questions

Which strategy proposed seems the most promising to you? Why?

Are there other strategies that can be used to optimize mathematical learning?

Thank you!

Any questions?

You may contact the mathematics programs team at:

FGJ-math@education.gouv.qc.ca

Bibliography

- Ministère de l'Éducation, du Loisir et du Sport (MELS) (2012). *New Approaches, New Solutions in Mathematics. Fostering Student Success in Disadvantaged Areas*. Gouvernement du Québec. http://www.education.gouv.qc.ca/fileadmin/site_web/documents/education/adaptation-scolaire-services-comp/SIAA_Math_reference_AN.pdf

- Ministère de l'Éducation du Québec (MEQ) (2006). *Programme de formation de l'école québécoise*. Gouvernement du Québec. http://www.education.gouv.qc.ca/fileadmin/site_web/documents/education/jeunes/pfeq/PFEQ_mathematique-primaire_EN.pdf

- Ministère de l'Éducation (MEQ) (2009). *Progression of Learning – Mathematics - Elementary*. Gouvernement du Québec. http://www.education.gouv.qc.ca/fileadmin/site_web/documents/education/jeunes/pfeq/PDA_PFEQ_mathematique-primaire_2009_EN.pdf

- Picard, C. (2018). *Enseigner la résolution de problèmes : accompagner les élèves de 5 à 12 ans dans le développement de la compétence à résoudre des problèmes*. Chenelière Éducation.

- Roy, George J., et al. "Linking Factors and Multiples to Algebraic Reasoning." *The Mathematics Teacher*, vol. 113, no. 3, National Council of Teachers of Mathematics, 2020, pp. 216–22.

- The Centre for Education in Mathematics and Computing (2021). *Problem of the Week – Problem C – Playing with Disks*. University of Waterloo.

- Van de Walle, J., Karp,K.S., Lovin, L.H. & Bay-Williams, J.M. (2018). *Teaching Student-Centered Mathematics : Developmentally Appropriate Instruction for Grades 3-5* (Vol.2, 3rd edition). Pearson.