Optimizing the Development of the Mathematical Competencies to Help Students Make Sense of the Mathematics

Secondary Cycle One

Fall 2021
Direction de la formation générale des jeunes
Ministère de l'Éducation

Québec

Welcome!

- **Geneviève Dupré**, Coordinator, Mathematics program
- **Raymond Nolin**, Elementary school mathematics teacher (loan of service)
- **Esther Veilleux**, Secondary school mathematics teacher (loan of service)

You are a(n) . . .

Teacher Cycle One	Teacher Cycle Two	Teacher Elementary	Education Consultant

Other duties

Objectives of the meeting

- Provide a clear explanation of intradisciplinary links (between competencies and between branches of mathematics) and interdisciplinary links.
- Explain how the development of the competencies and mathematical learning can be optimized.
- Provide examples of learning activities that foster the development of the competencies and make learning more meaningful.

Presentation outline

1. Connections between the competencies
2. Connections between the branches of mathematics
3. Mathematics in everyday life
4. Teaching strategies
5. Reflection questions

To foster reflection . . .

Students do not learn simply by being told.

They must experience cognitive dissonance,
which is central to learning mathematics.

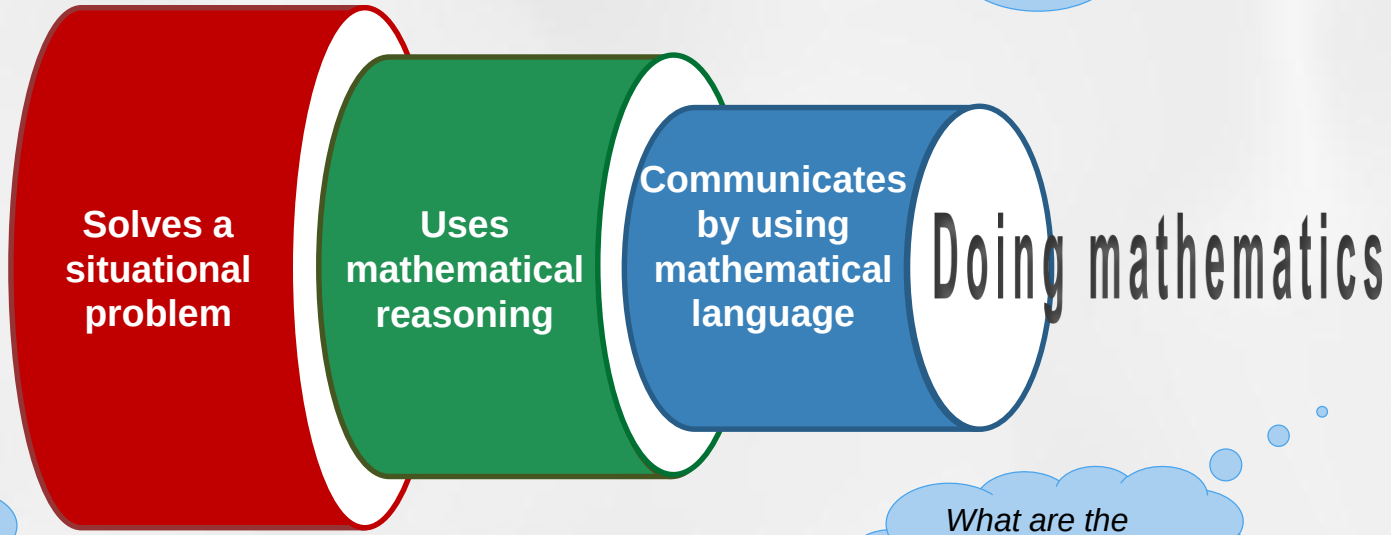
– Picard, 2018
[free translation]

Video on mathematical competencies

What do students need to learn?

What makes a person skilled in mathematics?

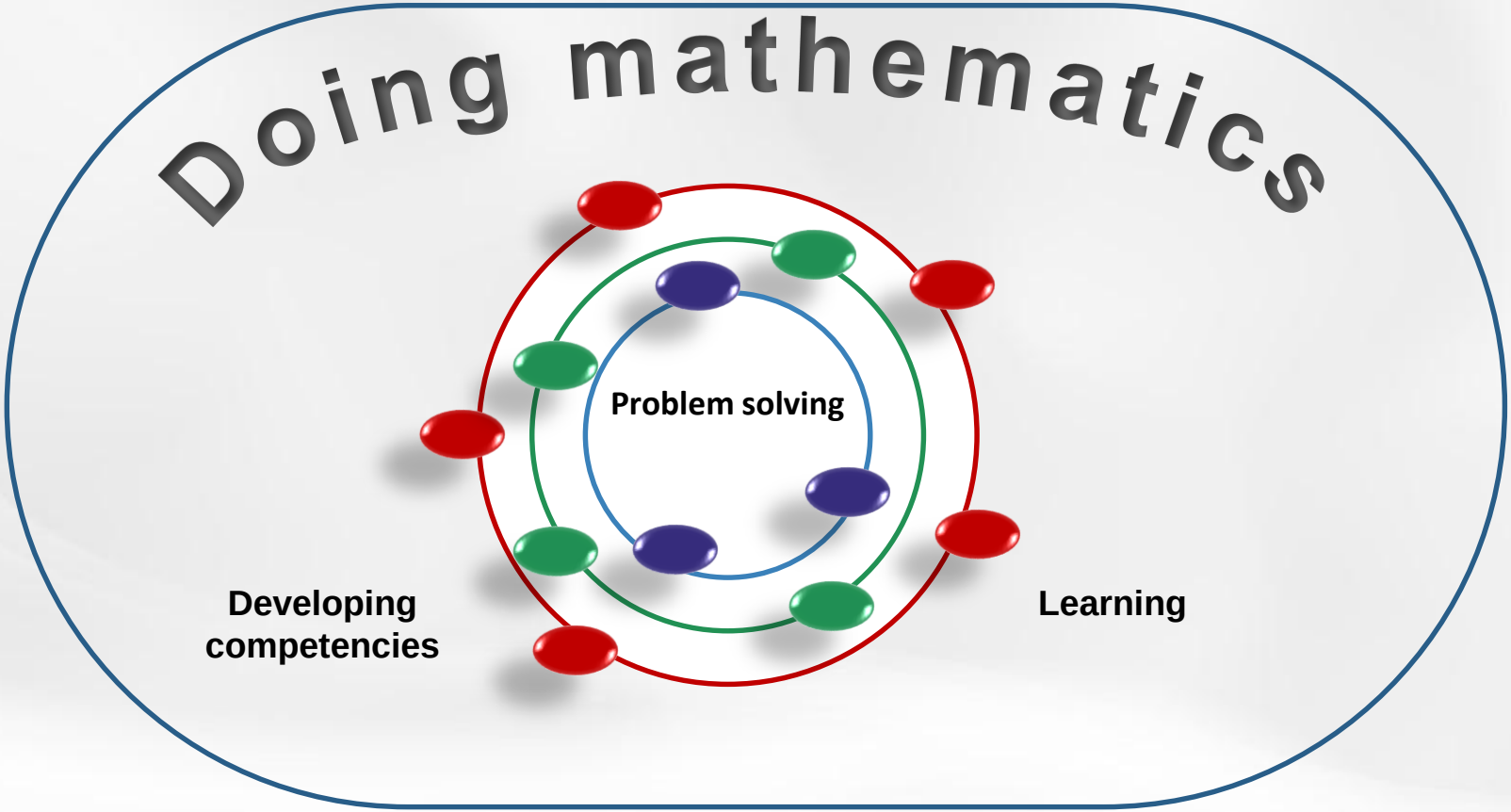
Developing the competencies in synergy



What does doing mathematics mean?

What are the learning objectives?

Connections between the competencies



Learning objectives

What are the learning objectives?

- To learn to solve mathematical problems.
- To use various problem-solving strategies.
- To use different types of representations.
- To use mathematical reasoning.
- To develop fluency and flexibility.
- To discover and reapply various concepts and processes.
- To use mathematical language to explain a procedure and solution(s).

Beginning of learning process

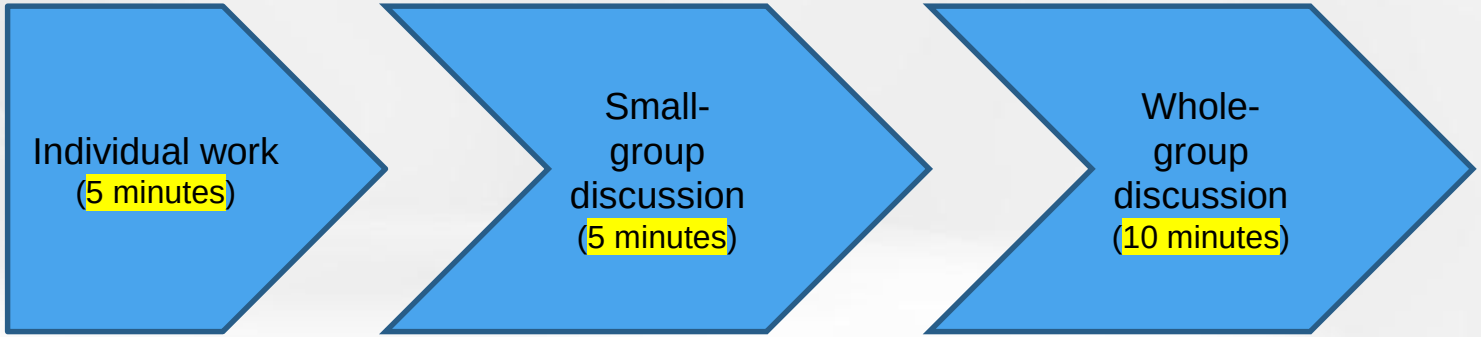
End of learning process

Example of a learning activity

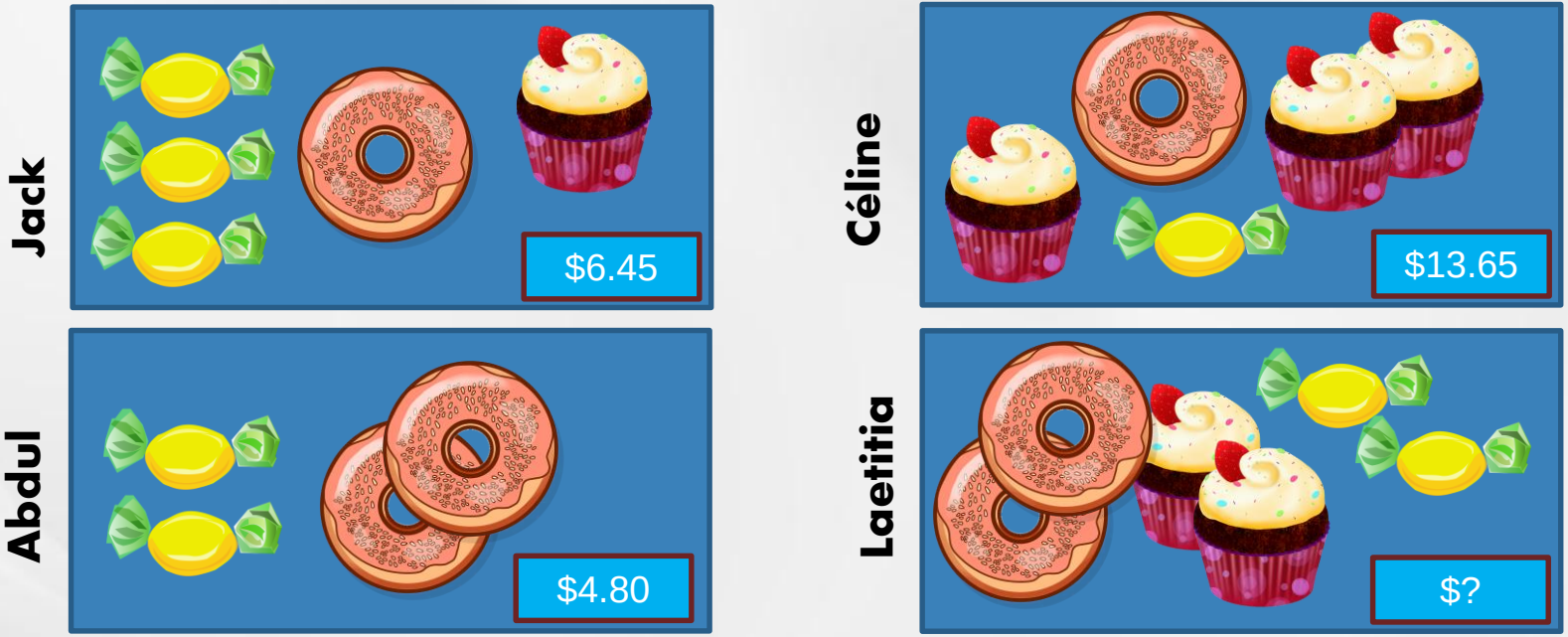
- Objective of the activity:

To illustrate how problem solving helps to develop the three mathematical competencies.

- Activity format: 5–10–15



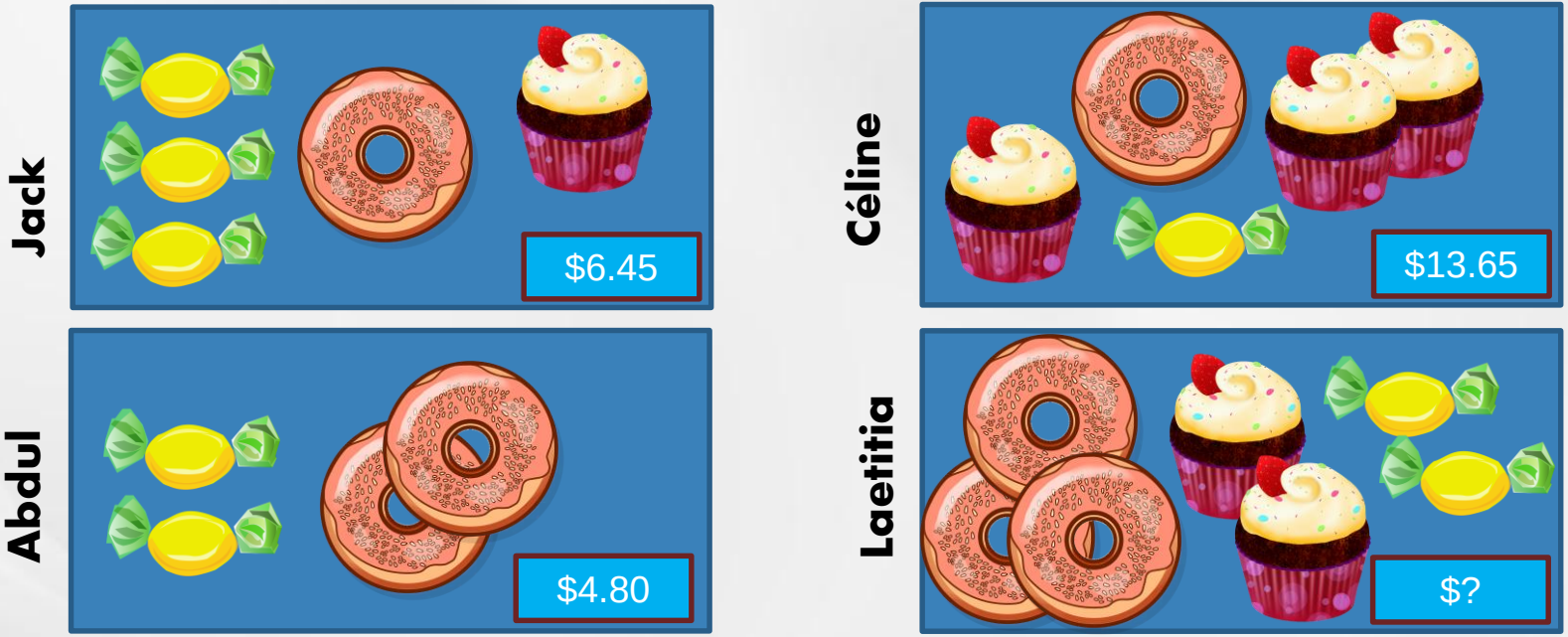
Example of a learning activity



How much will Laetitia pay?

Source: Translated and adapted from the problem “Dites-le avec des fleurs” from the magazine *Math-École*, no. 156, January 1993, 20.

Example of a learning activity



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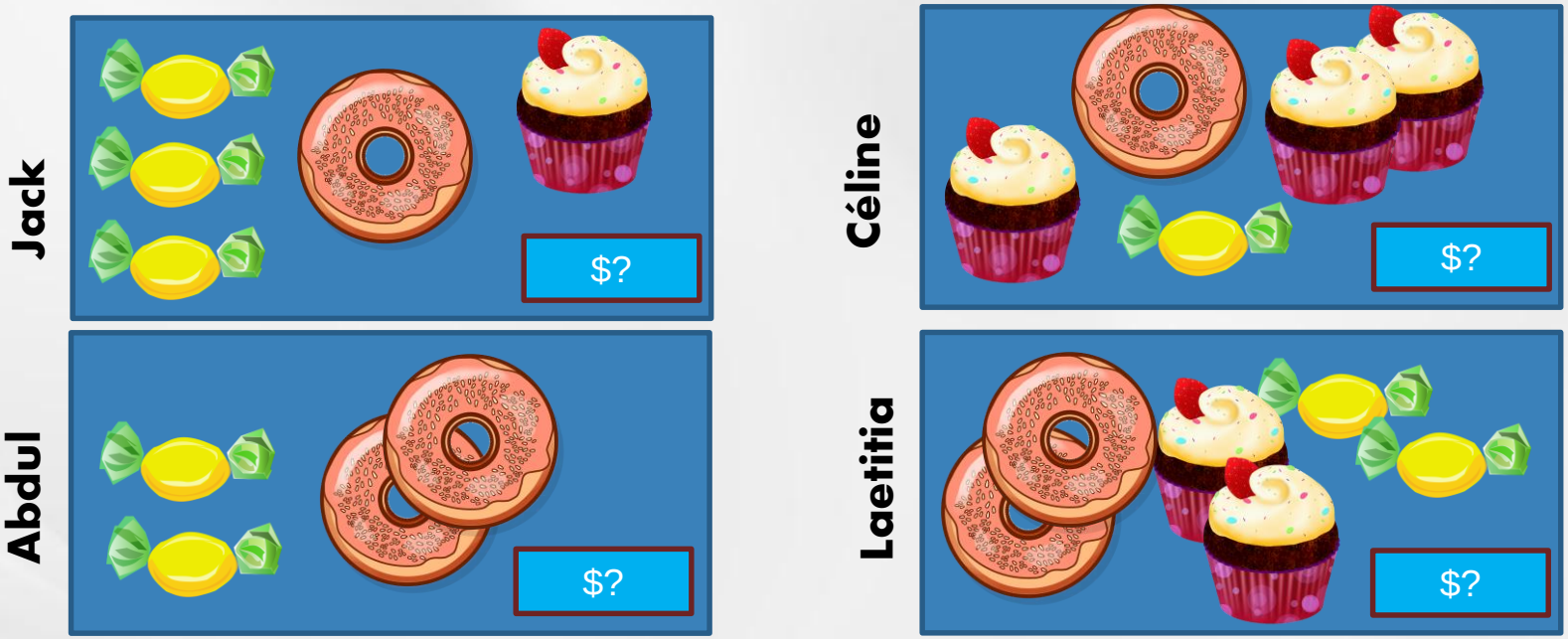
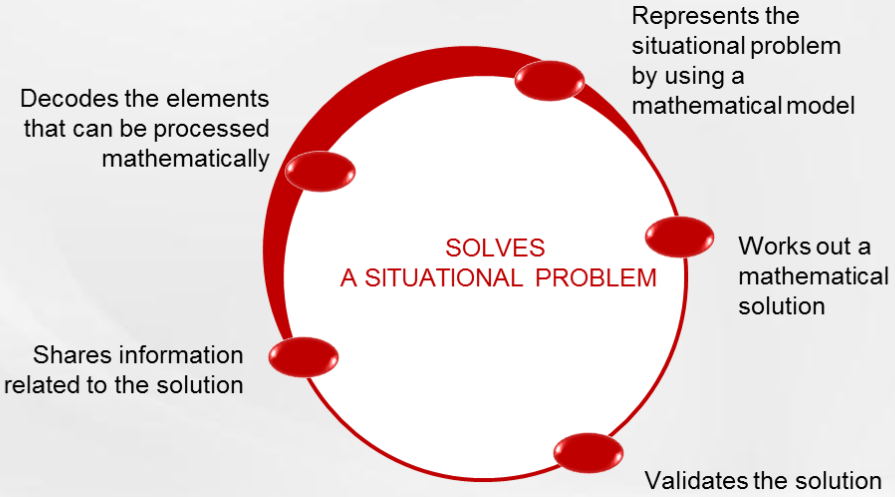


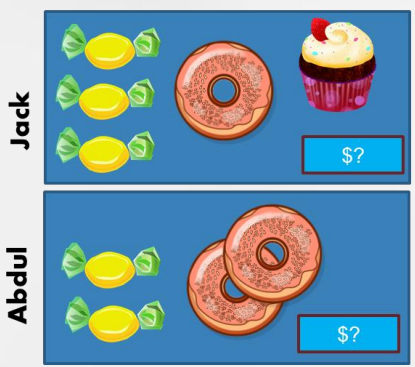
We know that the price of one doughnut is 3/5 of the price of one muffin and that the price of one candy is 4% of the price of one muffin. How much will each person pay if the total amount of the bill is $37.20?

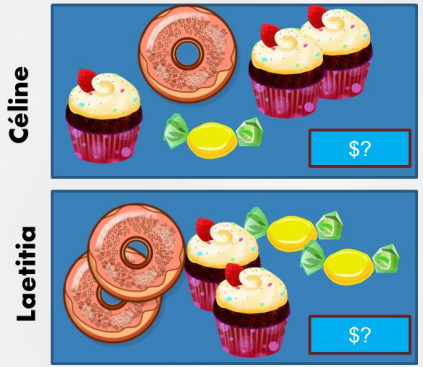
Source: Translated and adapted from the problem “Dites-le avec des fleurs” from the magazine *Math-École*, no. 156, January 1993, 20.

Example of a learning activity

Objective of the activity



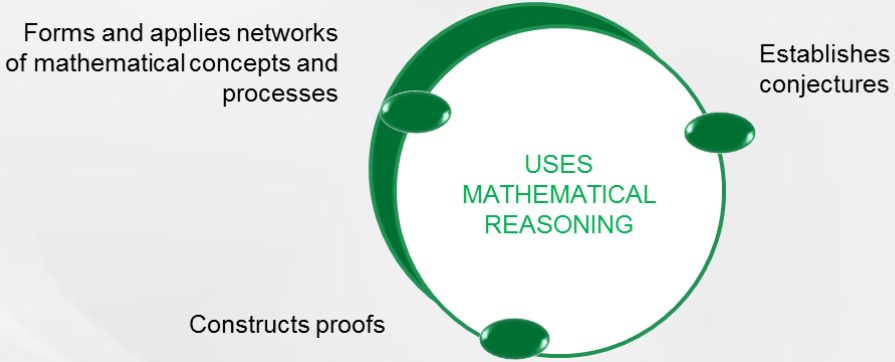


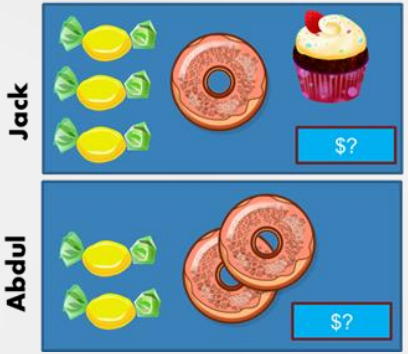


QEP, Secondary Cycle One, 2006, 241.

Example of a learning activity

Objective of the activity

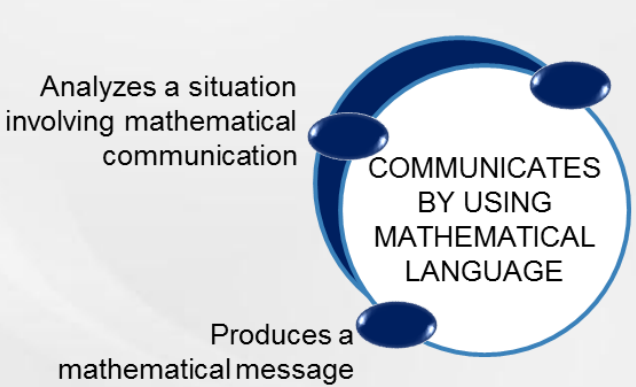


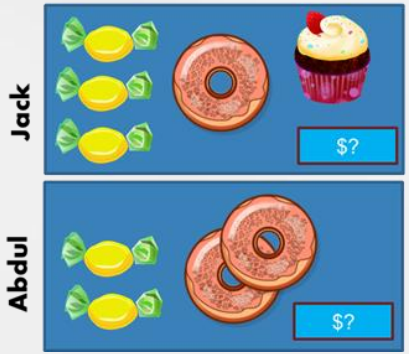


QEP, Secondary Cycle One, 2006, 203.

Example of a learning activity

Objective of the activity

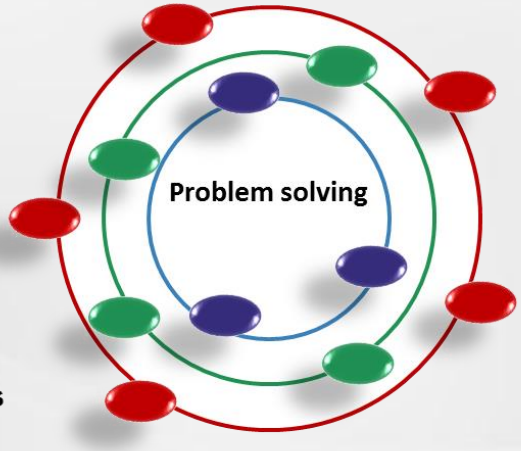




QEP, Secondary Cycle One, 2006, 205.

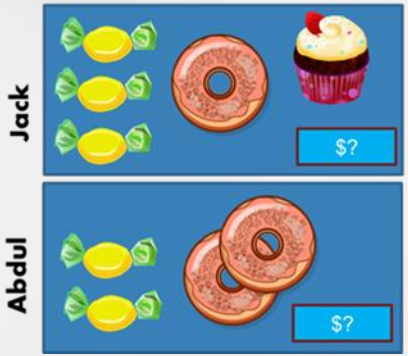
Example of a learning activity

Objective of the activity



Developing competencies

Learning



Another example of a learning activity

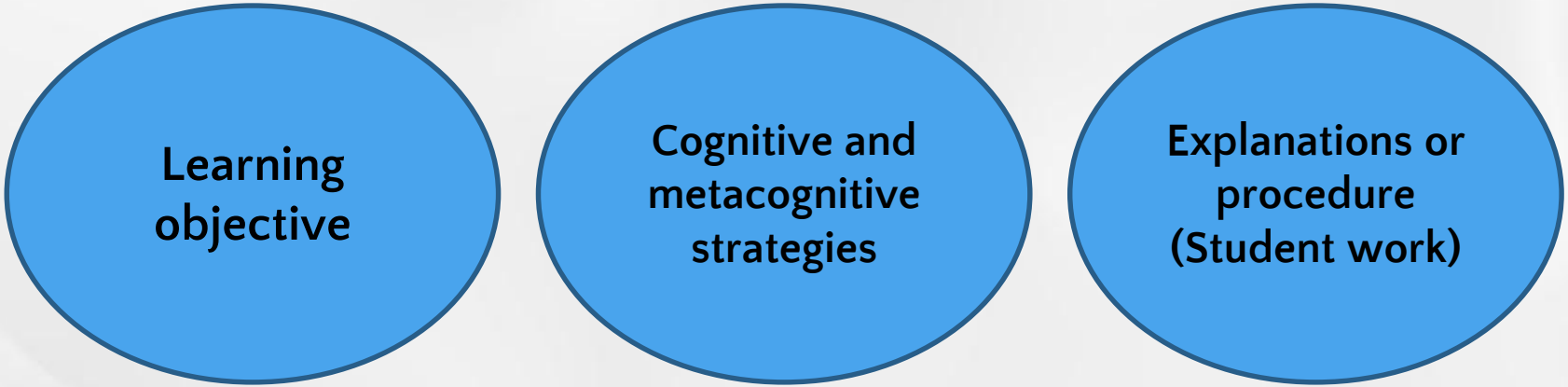
- **Using an application situation, start by asking only the open-ended question at the end of a problem.**

- *How much will it cost to paint the exterior of the concrete bulding recently built in Québec City?*

→ *What information do I need?*

→ *What information must I find to help me choose the right painting company?*

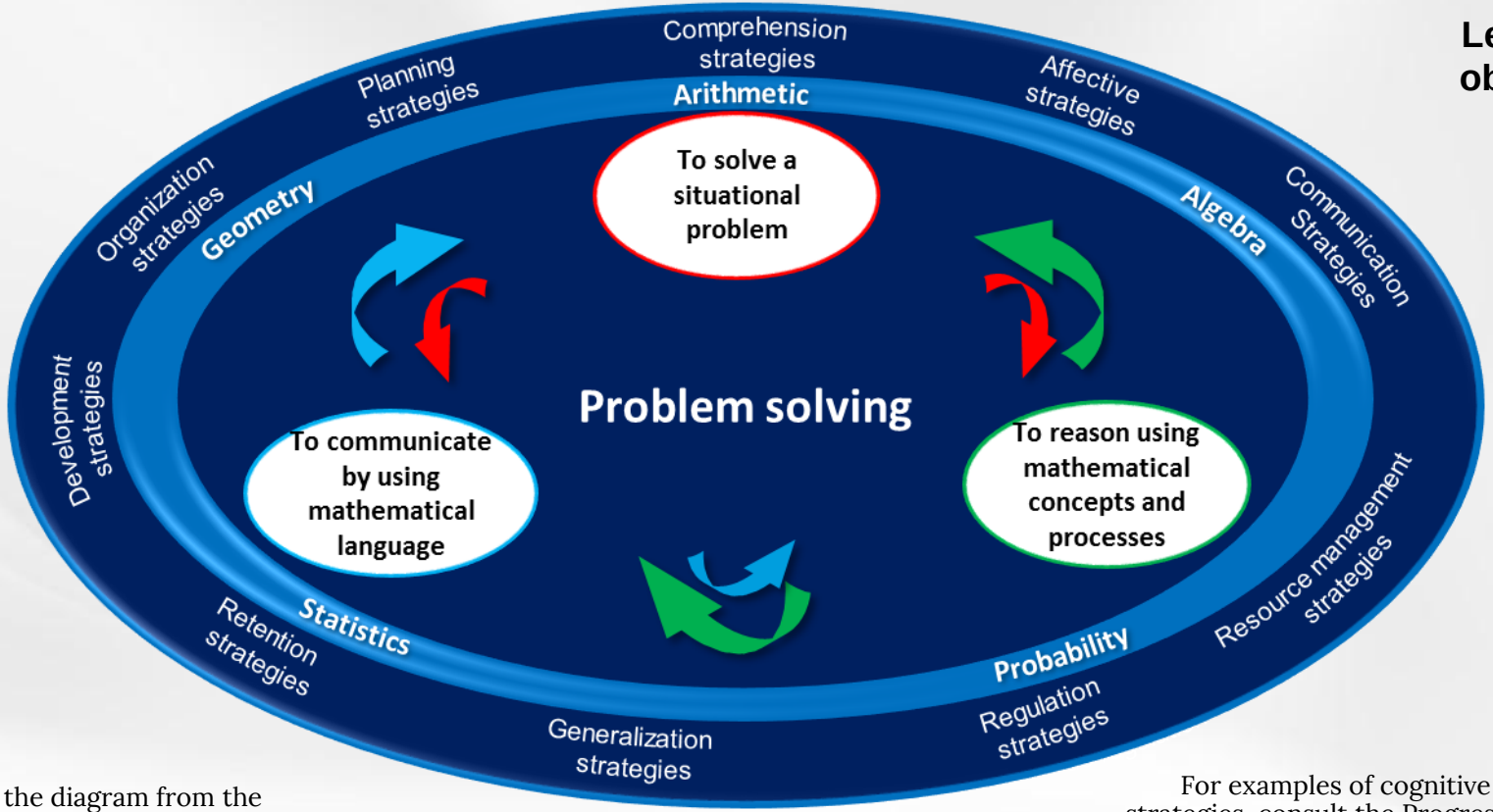
Asking the students questions



Optimizing learning

Connections between the competencies

Learning objective



Based on the diagram from the *Quebec Education Program* (QEP), Elementary, 2006, 141.

For examples of cognitive and metacognitive strategies, consult the Progression of Learning in Secondary School – Mathematics, 2016, pp. 42-44.

Encouraging reflection . . .

"The best we can do for students is to have them ask the right questions."

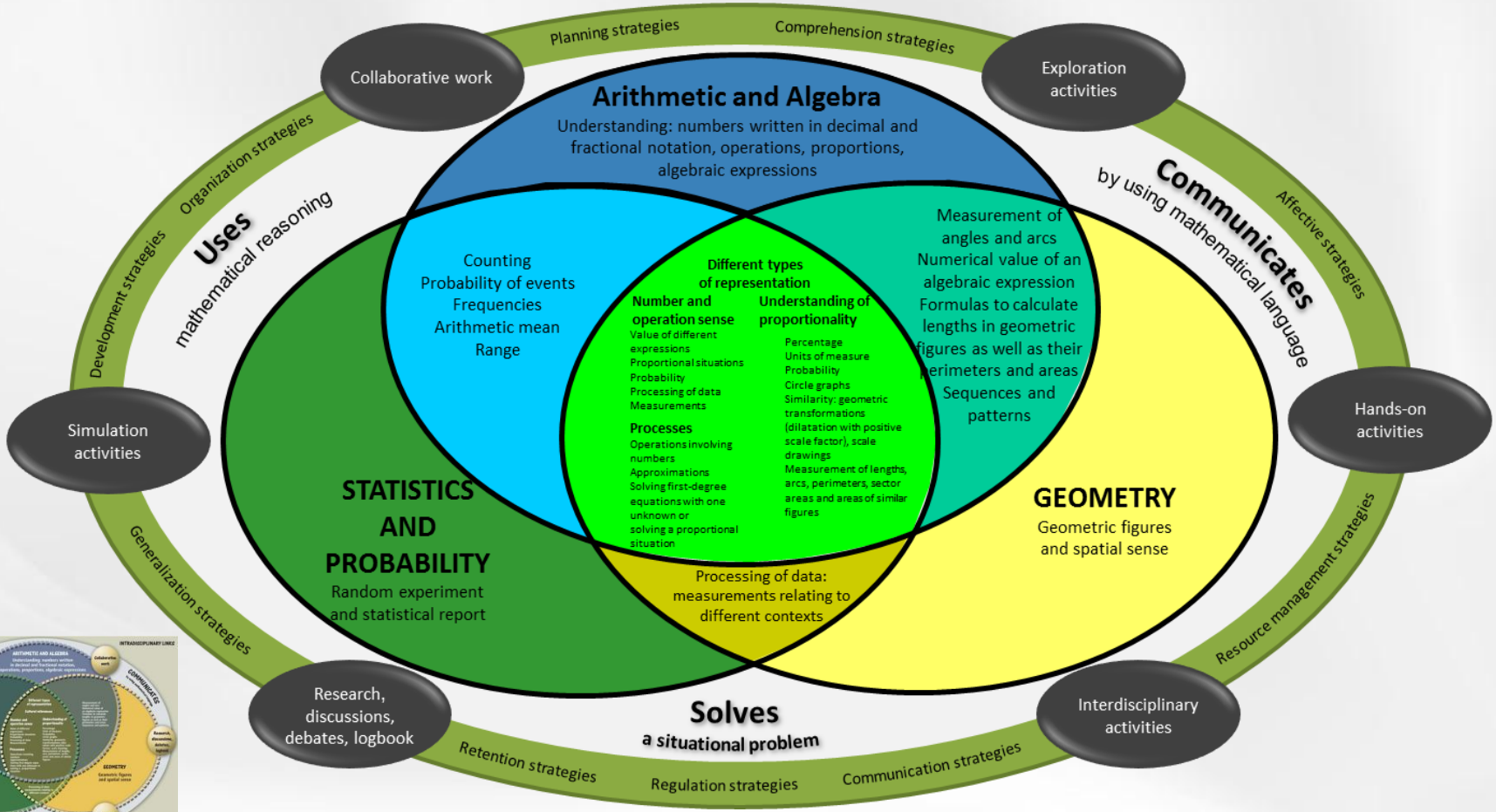
– Cantor, 2002, cited in OME, 2004

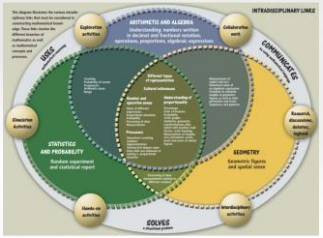
For more information about avenues for discussion, see the tables in each section of the document New Approaches, New Solutions in Mathematics. Fostering Student Success in Disadvantaged Areas.

2 Connections between the branches of mathematics

- The decompartmentalization of the branches of mathematics:
  - Helps students develop a greater understanding of concepts as well as greater flexibility and fluency
  - Makes the mathematical learning more meaningful
  - Helps students develop mathematical thinking as a whole
  - Optimizes learning by helping students make connections between mathematical concepts and processes

Intradisciplinary links



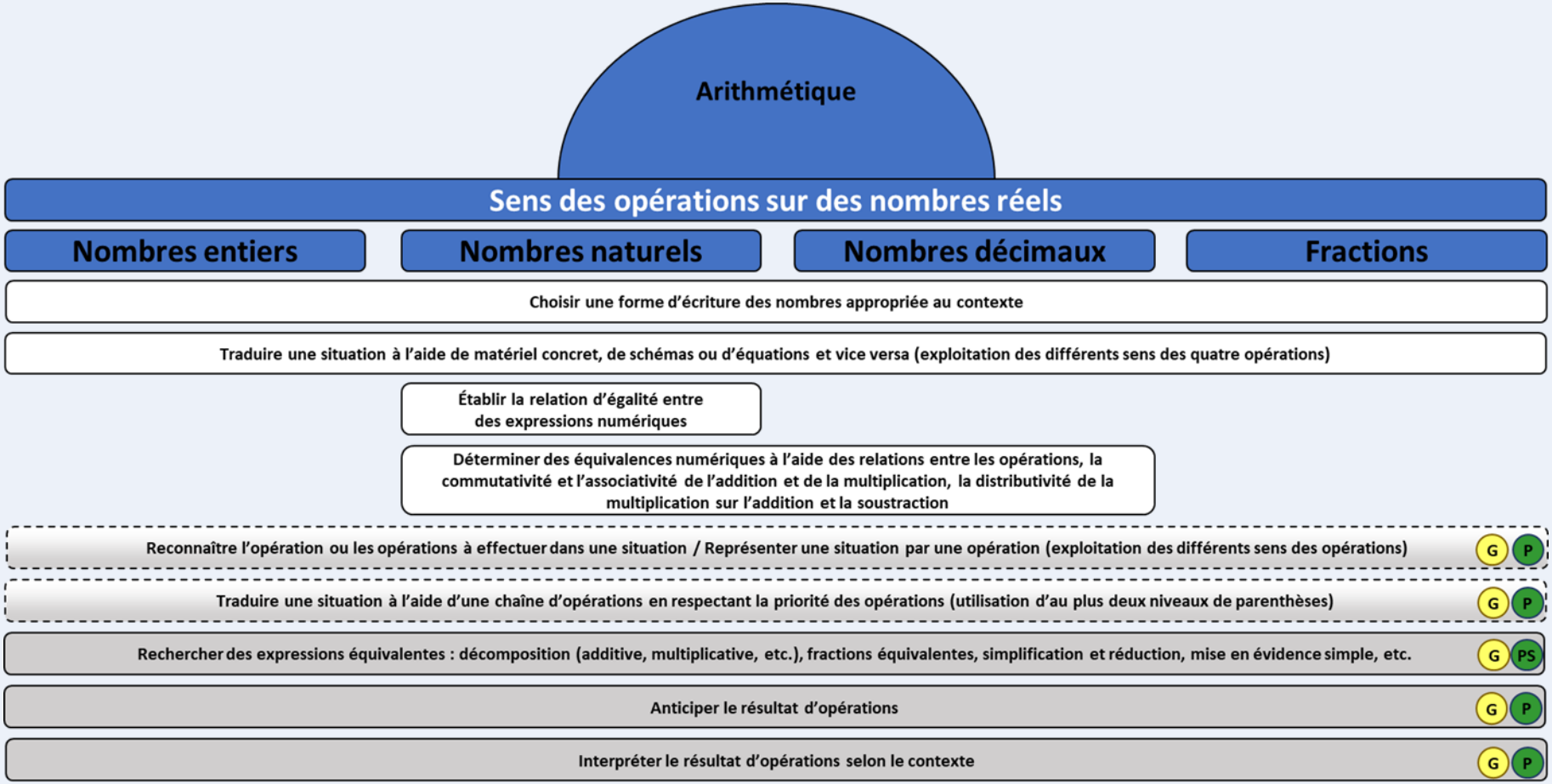


Source: QEP, Secondary Cycle One, 207.

Connections between the branches of mathematics

- A problem's context can sometimes help students discover, reapply or deepen their knowledge of elements associated with various branches of mathematics.

- Each branch of mathematics can provide the context needed to make mathematical learning more meaningful:
  - Geometry
  - Statistics
  - Probability



Aide-mémoire du programme d'étude en mathématique du 1er cycle du secondaire. (English version forthcoming)

Géométrie

Analyse de situations faisant appel à des mesures

Masses

- Choisir l'unité de mesure de masse appropriée au contexte
- Estimer et mesurer des masses à l'aide d'unités conventionnelles : gramme, kilogramme
- Établir des relations entre les unités de mesure de masse

Temps

- Choisir l'unité de mesure de temps appropriée au contexte
- Estimer et mesurer le temps à l'aide d'unités conventionnelles
- Établir des relations entre les unités de mesure de temps : seconde, minute, heure, jour, cycle quotidien, cycle hebdomadaire, cycle annuel
- Distinguer durée et position dans le temps (AA)

Angles

- Comparer des angles : angle aigu, angle droit, angle obtus
- Estimer et mesurer des angles en degrés
- Rechercher des angles manquants d'un triangle à partir de propriétés de figures et de relations (AA)
- Caractériser différents types d'angles : complémentaires, supplémentaires, opposés par le sommet, alternes-internes, alternes-externes et correspondants (AA)
- Rechercher des mesures d'angles en utilisant les propriétés des angles : complémentaires, supplémentaires, opposés par le sommet, alternes-internes, alternes-externes et correspondants (AA)
- Justifier des affirmations à partir de définitions ou de propriétés associées aux angles et à leurs mesures (AA)
- Rechercher des mesures manquantes d'angles au centre et d'arcs en degrés à partir des propriétés de figures et des relations (AA)

Longueurs

- Choisir l'unité de mesure de longueur appropriée au contexte
- Estimer et mesurer les dimensions d'un objet à l'aide d'unités conventionnelles : millimètre, centimètre, décimètre, mètre et kilomètre
- Établir des relations entre les unités de mesure de longueur : millimètre, centimètre, décimètre, mètre et kilomètre
- Établir des relations entre les mesures de longueur du système international (SI) (Ar)
- Construire les relations permettant de calculer le périmètre ou la circonférence de figures (AA)
- Justifier des affirmations relatives à des mesures de longueur (AA)
- Rechercher, à partir de propriétés des figures et des relations, les mesures manquantes :
  - périmètre de figures planes (primaire)
  - mesure d'un segment d'une figure plane, circonférence, rayon, diamètre, longueur d'un arc, mesure d'un segment provenant d'une isométrie ou d'une similitude (1re-2e sec.)
  - mesure de segments ou de périmètres issus de figures équivalentes (3e sec.) (AA)

Aide-mémoire du programme d'étude en mathématique du 1er cycle du secondaire. (English version forthcoming)

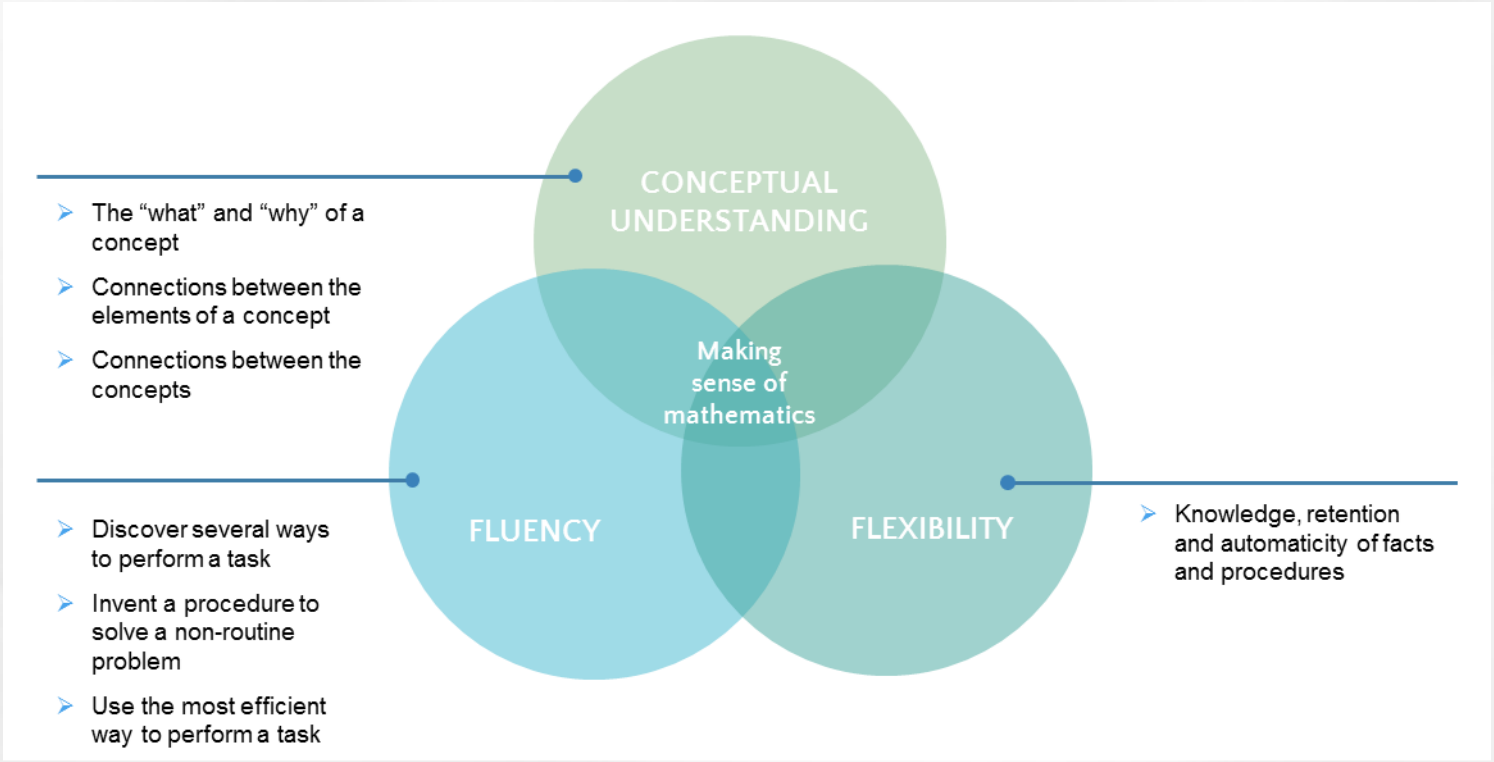
These diagrams are available on the website of the Ministère de l'Éducation under ***Aide-mémoire du programme d'études en mathématique*** (English forthcoming).

They are included among the documents related to the following training and information sessions, on the Secondary Mathematics page:

- ***Courses of action for a realistic and harmonized implementation of the mathematics programs***

- ***Optimizing the Development of the Mathematical Competencies to Help Students Make Sense of the Mathematics.***

Fostering flexibility and fluency in students



Source: Référentiel d'intervention en mathématique, (MEES, 2019) (English version forthcoming)

An example of a learning activity

- **Probability and statistics in sports**

How to calculate the mean, limits (bias) and maximum or minimum speed; how to compare the performances of athletes, and so much more!

- *Ranking teams in a tournament according to the number of matches, points and wins.*
- *Make a list of the records established in a sport.*
- *Calculate the probability that an athlete will win a match in given conditions.*
- *Determine the probability of winning or losing.*

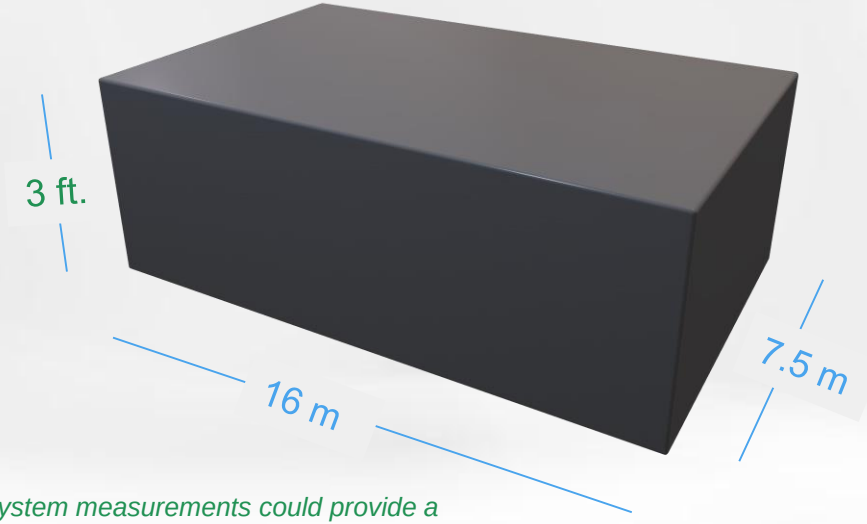
Numbers expressed in different ways: fraction, %, 3 wins and 2 losses

Different units of measure: m/s, km/h, points, etc.

Authentic contexts

Another example of a learning activity

- How much soil must I buy from the garden centre if I want to add 10 centimetres (4 inches) of soil to my raised garden bed? Would I save money by buying a truck load of soil instead of bags of soil?



$1.99 for 25 litres

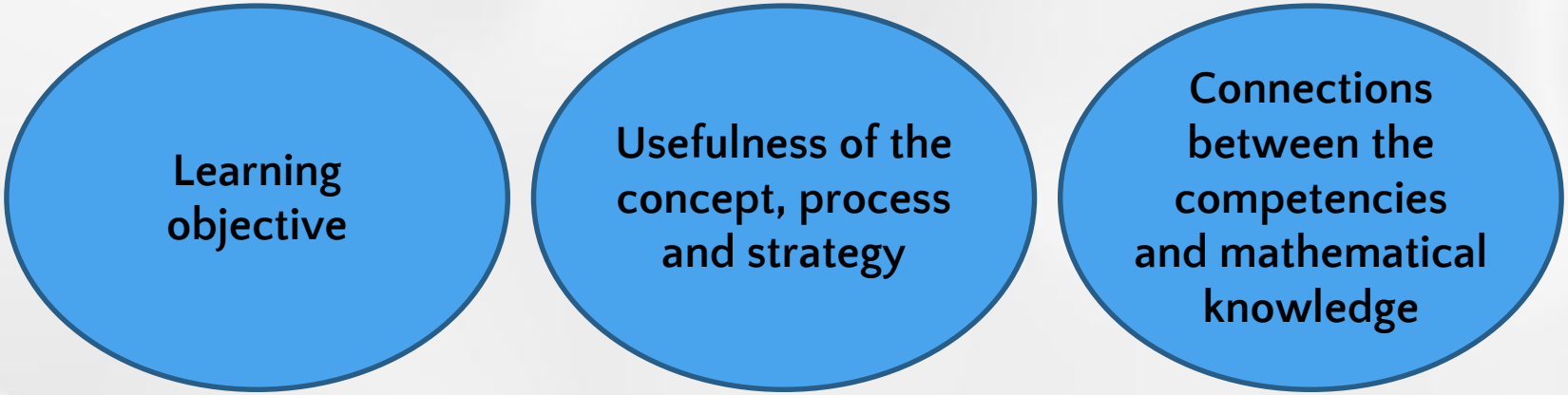
$56.00 per cubic meter
($46.00 per cubic yard)

Note: *Using imperial system measurements could provide a form of differentiation for students who are more advanced*

Other examples of learning activities

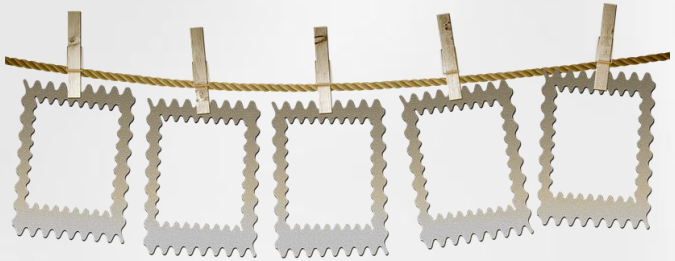
- *Using Environment Canada's website, have your students explore the many different mathematical situations that can be modelled with statistical data that is updated daily.*
- *Maximize the amount of material, paper, wrapping paper or sheet of aluminum that will be used to carry out a project.*
- *Discover how probabilities influence various daily decisions.*
- *Reproduce a room or building to scale.*
  (*See example: Minecraft Education en MST. [In French only]*)

Asking the students questions



Optimizing learning

An example of a learning activity



- **Clothesline Math**

This highly effective interactive tool facilitates transfers between the various branches of mathematics.

It can be used to help students understand and manipulate real numbers, proportions, fractions, percentages, algebra, the solving of algebraic equations, and much more.

For more information on Clothesline Math, view the recording of the networking session with Frédéric Ouellet, mathematics teacher: https://youtu.be/ieuG_eSck10. (In French only)

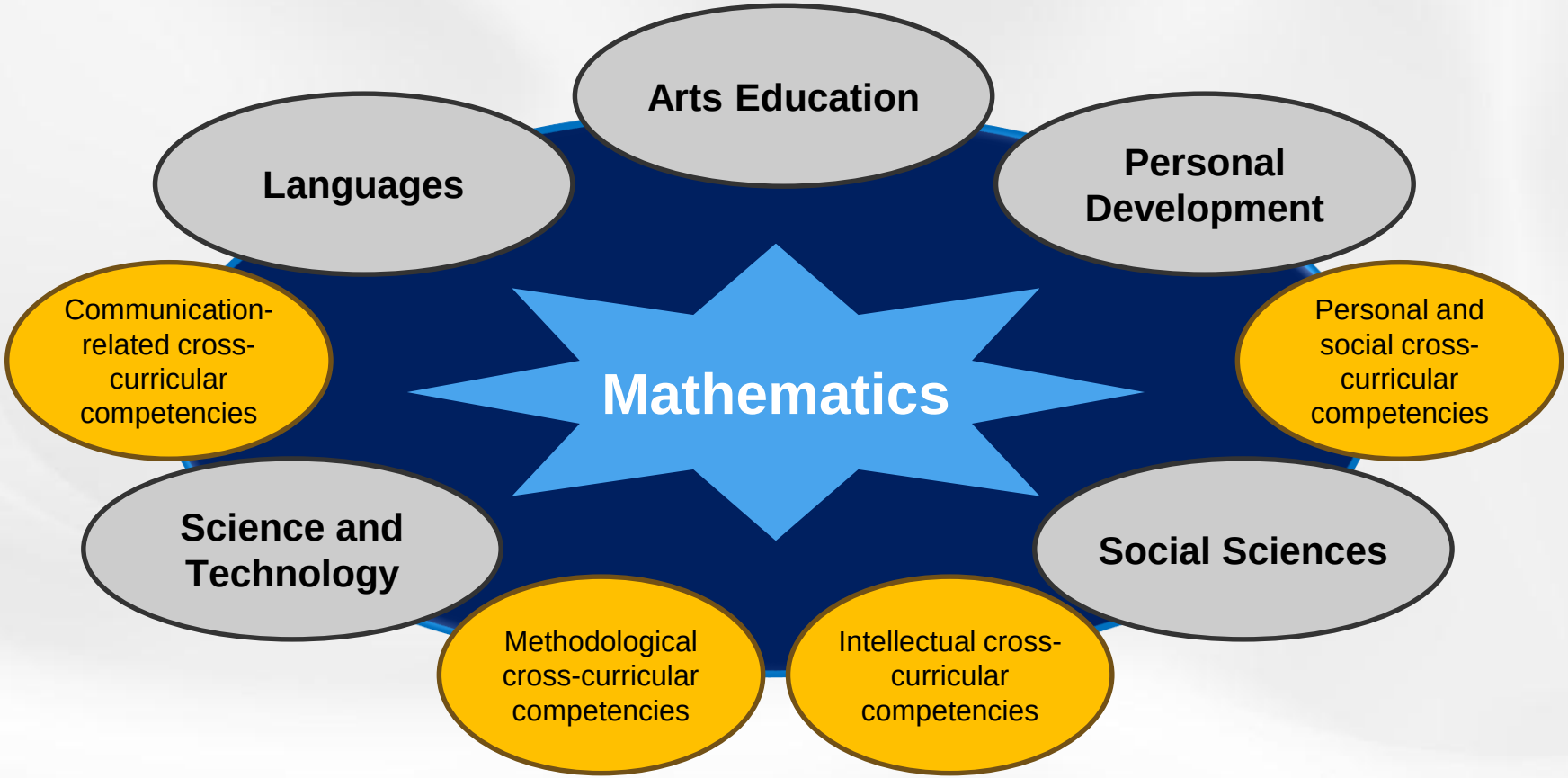
To foster reflection . . .

Students can come to realize that they are capable of making sense of mathematics. To teach effectively means to engage students at their level, thus enabling them to come up with or assimilate new ideas that will help them make sense of mathematics.

– Van de Walle and Lovin, 2007

[Free translation]

3 Mathematics in everyday life



An example of a learning activity

- **Electric cars**
  - How do you plan a trip in an electric car?
  - Study, understand and predict the car's range, its autonomy and the number of charges it needs.
  - Understand and convert the units of measure.

Another example of a learning activity

Nature provides us with an infinite number of possibilities to work on spatial sense. Why not be inspired by natural contexts to create problems or projects for students?

Using rocks and minerals to study polyhedrons:

- Create crystals that have symmetry
- Create pursuit curves
- Reproduce a building to scale: [see an example in Minecraft Education en MST] [In French only]

Mathematics all around us

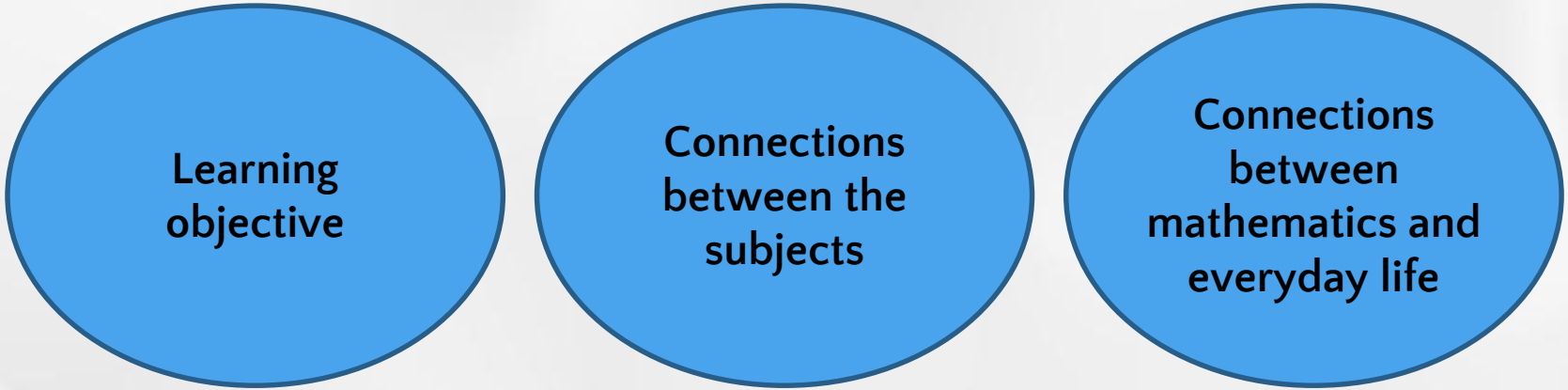
- Make connections with the competencies of other subjects to make learning more meaningful.
- Learn the terminology of the various subjects, become aware of the connections between the different types of learning and the different types of representation.
- Several of these interdisciplinary links can also be used to explore various **cultural references**.

For more information on cultural references, view the presentation recording (in French only) on the cultural approach to secondary school mathematics on the website of the Ministère de l'Éducation. (Reference documents available in English)

Other examples of learning activities

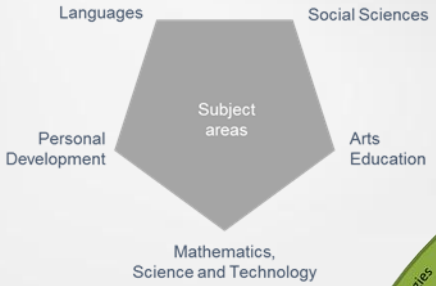
- Think about how an access ramp should be built to address the specific needs of people with reduced mobility.
- Conduct surveys to address current needs or important issues for students.
- Predict the tides and compare them from one region to another. (Determine the times at which it is not safe to be in a certain location on the beach.)
- Design a timeline, create a number line with large numbers or work with numbers related to the arts.

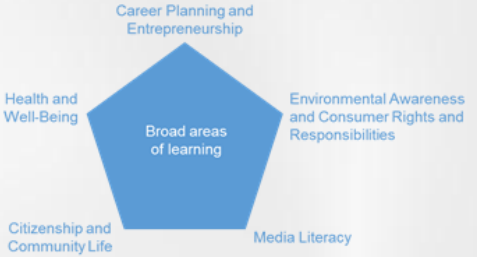
Asking the students questions

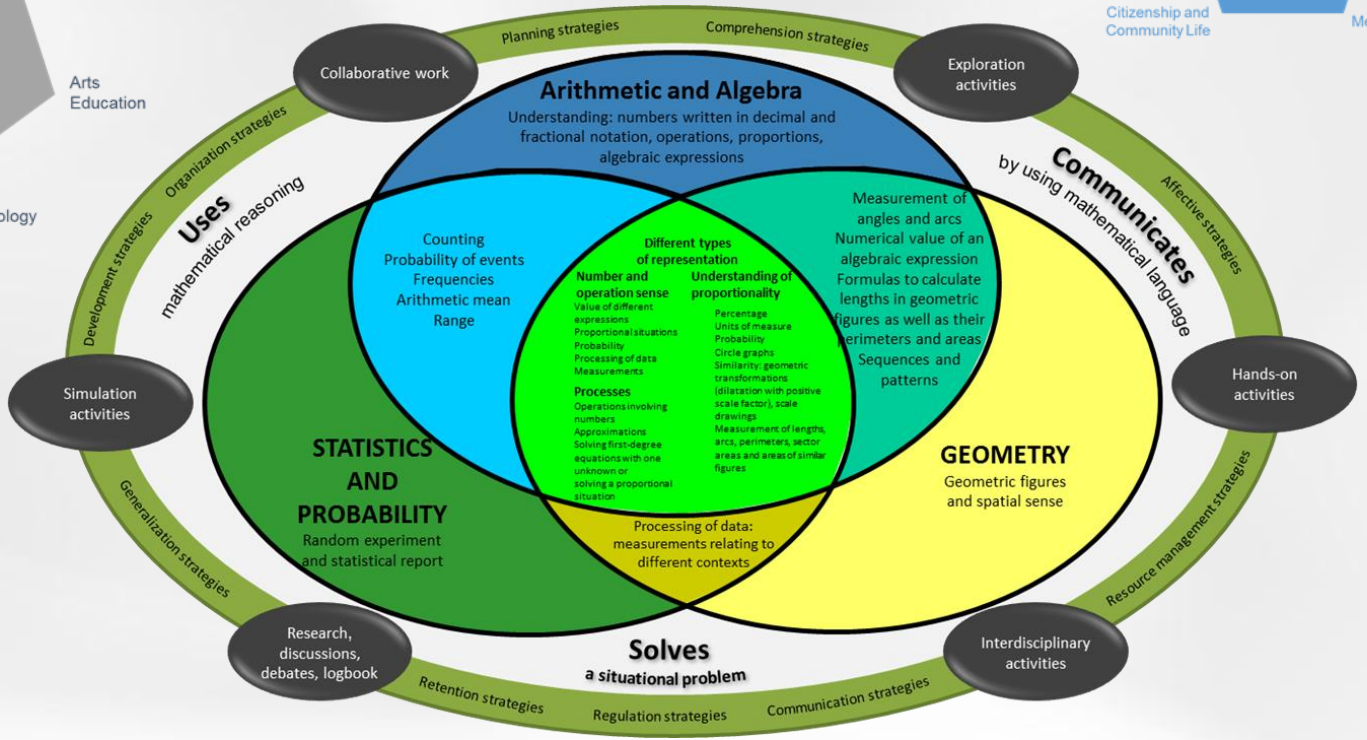


Optimizing learning

All in one . . .







Source: QEP, Secondary Cycle One, 207.

4 Teaching strategies that foster the optimization of learning

Problem solving

Mathematical discussions

Classroom climate

Various types of reasoning

Teaching strategies that foster the optimization of learning

Frequent and meaningful feedback

Mathematical routines

Modelling

Student's portrait

Teaching strategies that foster the optimization of learning

Engaging and concrete activities

Common reference documents

Concrete examples based on everyday life

Error as a learning opportunity

A variety of resources

A variety of resources

Manipulatives

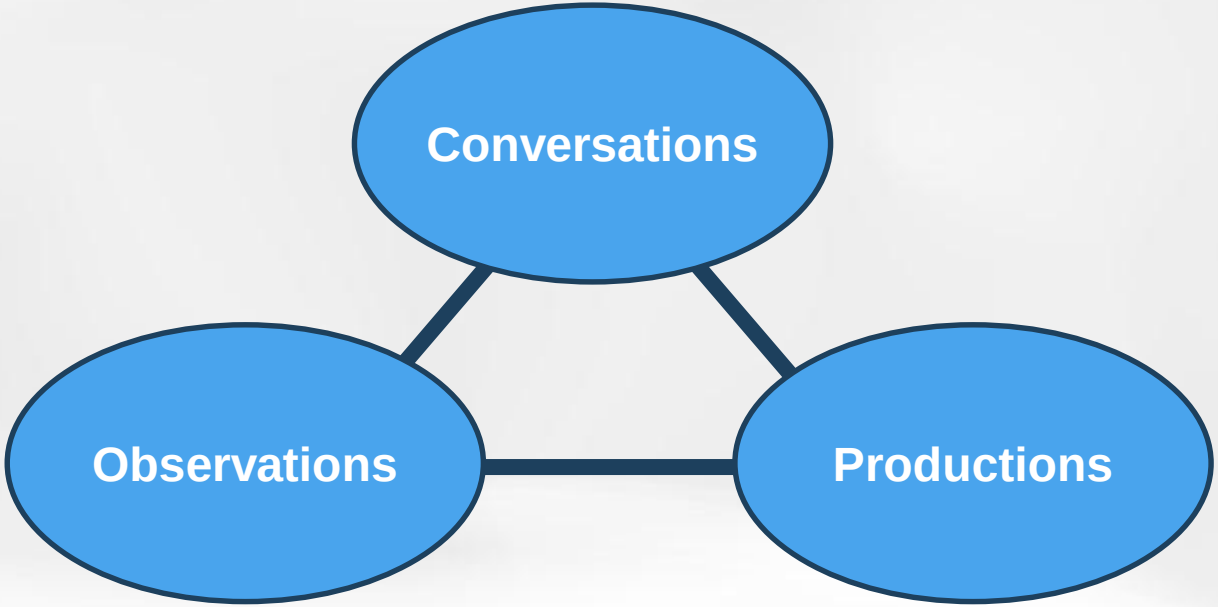
Ecological approach

Authentic material

ICT and programming

Teaching strategies that foster the optimization of learning

- Provide varied feedback:



Food for thought . . .

No single approach can guarantee success for all students. Teachers' practices must comprise a balanced mixture of different approaches, teaching strategies and organizational arrangements.

– UNESCO, 2000, from *New Approaches, New Solutions*

5 Reflection questions

Which strategy proposed seems the most promising to you? Why?

Are there other strategies that can be used to optimize mathematical learning?

Thank you!

Any questions?

You may contact the mathematics programs team at:

FGJ-math@education.gouv.qc.ca

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