Ministère de l'Éducation et de l'Enseignement supérieur
Québec

Clarifications regarding the types of reasoning to be used in secondary school mathematics

See below for the answers to the chat questions asked during the information session on April 26, 2018, as well as additional information.

The video recording of the meeting is available (in French only) on the website of the Ministère de l'Éducation et de l'Enseignement supérieur.

Questions and answers

1. **Question: Is a conjecture a form of reasoning?**
   Answer: "The term *conjecture* refers to a statement that is thought to be true. The verb *to conjecture* means to have a sense that a statement is true and to try to show that it is true" (QEP, Cycle One, p. 195). Conjecturing tasks are thus one of the means through which students develop their reasoning abilities. Conjectures can be the starting point or culmination of a reasoning task. In the latter case, conjectures must be reasoned out before they are formulated.

2. **Question: What role do conjectures play in the curriculum and in the development of Competency 2?**
   Answer: Although only one of the three key features of Competency 2 refers explicitly to conjectures (*Makes conjectures*), they are central to the development of this competency. Moreover, another key feature also involves conjectures (*Constructs proofs*). The first subcomponent of this key feature reads as follows: "Uses different types of reasoning (e.g. inductive, deductive, analogical, proof by exhaustion, proof by contradiction) to clarify, validate, adjust or refute conjectures" (QEP, Cycle Two, p. 29). The curriculum states that "Using mathematical reasoning involves making conjectures and [assessing], justifying or refuting a proposition by applying an organized body of mathematical knowledge" (QEP, Cycle One, p. 200). In addition, "A situation involving applications entails one or more implicit or explicit conjectures (relationships, statements, opinions, conclusions, etc.) that must be discovered, explained, generalized, proved or refuted using mathematical knowledge" (QEP, Cycle Two, p. 26).

3. **Question: What type of reasoning is associated with proof by contradiction?**

   Answer: Proof by contradiction is also known as *reductio ad absurdum* (Latin for "reduction to absurdity"). This is a form of deductive reasoning, like proof by exhaustion (QEP, Cycle Two, p. 26).

4. **Question: Should the students choose the type of reasoning they will use or should the teacher require that a specific type of reasoning be used for a given task?**

   Answer: Reasoning tasks complement the learning process and help students develop a better understanding of concepts. Students should not be asked to carry out a task by using a specific type of reasoning. Instead, they should be presented with a variety of situations in which they can use different types of reasoning. In fact, students can use different forms of reasoning to answer the same question. When they learn through various situations that require them to reason rather than memorize a procedure, students acquire the ability to examine more complex questions. Encouraging students to share their reasoning and compare it with that of their peers will give them the opportunity to experiment with different approaches. It may be useful for the teacher to name the various types of reasoning during the learning process in order to familiarize the students with the terms that refer to the different possible approaches, but this vocabulary need not be memorized.

5. **Question: Is there a progression of expectations when it comes to formulating and validating a conjecture? Should Cycle Two students be assigned conjecturing tasks that require them to provide a proof to support their conjectures?**

   Answer: Yes, there is a progression of expectations with regard to the validation of a conjecture. By the end of Cycle One, students are expected to "try different approaches in order to determine whether they should confirm or refute their conjectures [and to] validate them either by basing each step of their solution on concepts, processes, rules or statements that they express in an organized manner, or by supplying counterexamples" (QEP, Cycle One, p. 203). By the end of Cycle Two, students are expected to make conjectures and confirm or refute them by using various types of reasoning that involve applying appropriate concepts and processes. They also validate their conjectures "by basing each step in their proof on concepts, processes, rules or postulates, which they express in an organized manner" (QEP, Cycle Two, p. 30). The curriculum also states that the ability to construct proofs implies that students can "distinguish between reasoning and a mathematical proof, which is the codified presentation of that reasoning. Writing out the proof is therefore the last step in the process of validating a conjecture. This written presentation can be described as an explanation or a formal proof, depending on the student's approach" (QEP, Cycle Two, p. 27). Students "learn how to better explain and organize their reasoning and how to fine-tune their supporting arguments. [They] gradually learn to construct rigorous proofs" (QEP, Cycle Two, p. 31). In the presentation of Competency 2, the wording of the key feature *Constructs proofs* in each cycle also shows a progression of expectations (QEP, Cycle One, p. 203, and Cycle Two, p. 29). The same applies to the wording of evaluation criteria 4 and 5.

6. **Question: Why is there a requirement that three different examples be used to support a conjecture?**

   Answer: When students use exemplification to arrive at a conjecture, the minimum number of examples to be provided depends on the situation. If students must formulate a conjecture regarding the relationship between two values, they will have to provide more than two examples to be able to determine whether there is a constant, proportional, exponential or other type of relationship between these values (see the third example in slide 20, or the example in slide 32 of the PowerPoint presentation). If students must formulate a conjecture regarding only one value, only two examples may be required in certain cases to formulate an observation (see the first example in slide 27 of the PowerPoint presentation). It should be noted that students may need a greater number of examples to support their conjecture, depending on the values they use in their examples.

7. **Question: I see that in slide 23 of the presentation, you have provided an example of a task involving the concepts of greatest common divisor (GCD) and least common multiple (LCM). Are the GCD and LCM compulsory concepts?**

   Answer: These are not compulsory concepts in the mathematics curriculum. However, they can be used for exploration purposes and applied, for example, to fraction-related work or problem-solving tasks.

8. **Question: In presenting certain examples, such as those in slides 25 and 32, you mentioned that students could use manipulatives to develop their reasoning. What do you mean by "manipulatives?"**

   Answer: The manipulatives provided for students (e.g. number lines, counters or similar materials, cubes, Cuisenaire rods or equivalent products) can be purchased or hand-made. Manipulatives can also consist of recycled materials (e.g. cardboard boxes, cardboard tubes from paper towel rolls, bottle caps, bread bag ties, pasta). Students can also use applications or dynamic geometry software to, among other things, generate a variety of examples or simulate a situation.

Additional information

Did you know that the terms “conjecture” or “to conjecture” appear 30 times in the Secondary Cycle One QEP and 118 times in the Secondary Cycle Two QEP?

Reference material:

- ☞ Secondary Cycle One Mathematics Curriculum
- ☞ Secondary Cycle Two Mathematics Curriculum

Information concerning the **types of reasoning related to the different branches of mathematics**:

- QEP, Cycle One, pp. 200-201
- QEP, Cycle Two, pp. 27-28

Principles of Euclidean geometry that can be discovered by students, or used to develop their reasoning abilties or to devise conjecturing tasks:

- QEP, Cycle One, p. 219

Information on the types of tasks to be explored through reasoning in the curriculum:

- QEP, Cycle One, pp. 209-219
- QEP, Cycle Two, pp. 122-130